HYBRID CHAOS SYNCHRONIZATION OF 4-D HYPERCHAOTIC QI AND JIA SYSTEMS BY ACTIVE NONLINEAR CONTROL

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ABSTRACT

This paper investigates the hybrid chaos synchronization of identical 4-D hyperchaotic Qi systems, 4-D identical hyperchaotic Jia systems and hybrid synchronization of 4-D hyperchaotic Qi and Jia systems. The hyperchaotic Qi system (2008) and hyperchaotic Jia system (2007) are important models of hyperchaotic systems. Hybrid synchronization of the 4-dimensional hyperchaotic systems addressed in this paper is achieved through complete synchronization of two pairs of states and anti-synchronization of the other two pairs of states of the underlying systems. Active nonlinear control is the method used for the hybrid synchronization of identical and different hyperchaotic Qi and Jia systems and the stability results have been established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed nonlinear control method is effective and convenient to achieve hybrid synchronization of the hyperchaotic Qi and Jia systems. Numerical simulations are presented to demonstrate the effectiveness of the proposed chaos synchronization schemes.

KEYWORDS

Active Nonlinear Control, Hybrid Synchronization, Hyperchaos, Hyperchaotic Qi System, Hyperchaotic Jia System.

1. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly called as the *butterfly effect* [1]. Chaos is an interesting nonlinear phenomenon and has been extensively and intensively studied in the last two decades [1-23]. Chaos theory has been applied in many scientific disciplines such as Mathematics, Computer Science, Microbiology, Biology, Ecology, Economics, Population Dynamics and Robotics.

In 1990, Pecora and Carroll [2] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical [3], chemical [4], ecological [5] systems, secure communications [6-7], etc.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism has been used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of synchronization is to use

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the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the seminal work by Pecora and Carroll [2], a variety of impressive approaches have been proposed for the synchronization of chaotic systems such as the sampled-data feedback synchronization method [8], OGY method [9], time-delay feedback method [10], backstepping method [11], adaptive design method [12], sliding mode control method [13], etc.

So far, many types of synchronization phenomenon have been presented such as complete synchronization [2], phase synchronization [5, 14], generalized synchronization [7, 15], anti-synchronization [16, 17], projective synchronization [18], generalized projective synchronization [19, 20], etc.

Complete synchronization (CS) is characterized by the equality of state variables evolving in time, while anti-synchronization (AS) is characterized by the disappearance of the sum of relevant variables evolving in time. Projective synchronization (PS) is characterized by the fact that the master and slave systems could be synchronized up to a scaling factor, whereas in generalized projective synchronization (GPS), the responses of the synchronized dynamical states synchronize up to a constant scaling matrix α. It is easy to see that the complete synchronization (CS) and anti-synchronization (AS) are special cases of the generalized projective synchronization (GPS) where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

In hybrid synchronization of chaotic systems [20], one part of the system is synchronized and the other part is anti-synchronized so that the complete synchronization (CS) and anti-synchronization (AS) coexist in the system. The coexistence of CS and AS is highly useful in secure communication and chaotic encryption schemes.

This paper is organized as follows. In Section 2, we derive results for the hybrid synchronization of identical hyperchaotic Qi systems ([22], 2008). In Section 3, we derive results for the hybrid synchronization of identical hyperchaotic Jia systems ([23], 2007). In Section 4, we derive results for the hybrid synchronization of non-identical hyperchaotic Qi and Jia systems. The nonlinear controllers are derived using Lyapunov stability theory for the hybrid synchronization of the two hyperchaotic systems. The proposed nonlinear control method is simple, effective and easy to implement in practical applications. Conclusions are contained in the final section.

2. HYBRID SYNCHRONIZATION OF IDENTICAL QI SYSTEMS

In this section, we consider the hybrid synchronization of identical hyperchaotic Qi systems [22]. Thus, we consider the master system as the hyperchaotic Qi dynamics described by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\
\dot{x}_3 &= -c x_3 - \varepsilon x_4 + x_1 x_2 \\
\dot{x}_4 &= -d x_4 + f x_3 + x_1 x_2
\end{aligned} \tag{1}$$

where $x_i \ (i = 1,2,3,4)$ are the state variables and $a, b, c, d, \varepsilon, f$ are positive constants.

When $a = 50,\ b = 24,\ c = 13,\ d = 8,\ \varepsilon = 33$ and $f = 30,$ the Qi system (1) is hyperchaotic (see Figure 1).

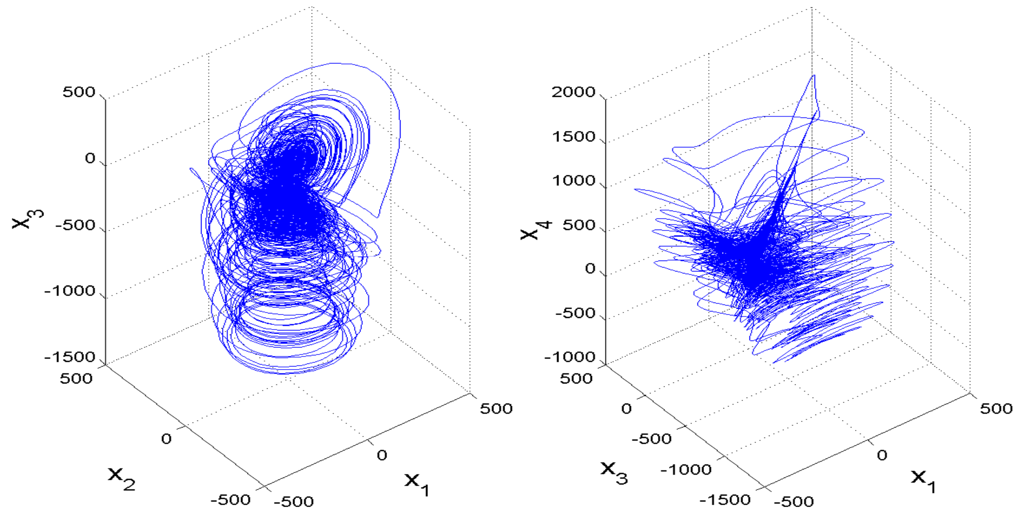

Figure 1. State Orbits of the Hyperchaotic Qi System (1)

We consider the hyperchaotic Qi dynamics also as the *slave* system, which is described by

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\
\dot{y}_2 &= b(y_1 + y_2) - y_1 y_3 + u_2 \\
\dot{y}_3 &= -c y_3 - \varepsilon y_4 + y_1 y_2 + u_3 \\
\dot{y}_4 &= -d y_4 + f y_3 + y_1 y_2 + u_4
\end{aligned} \tag{2}$$

where $y_i \ (i = 1, 2, 3, 4)$ are the state variables and $u_i \ (i = 1, 2, 3, 4)$ are the active controls.

For the hybrid synchronization of the identical hyperchaotic Qi systems (1) and (2), the synchronization errors are defined as

$$e_1 = y_1 - x_1, \ \ e_2 = y_2 + x_2, \ e_3 = y_3 - x_3, \ \ e_4 = y_4 + x_4 \tag{3}$$

From the error equations (3), it is clear that one part of the two hyperchaotic systems is completely synchronized (first and third states), while the other part is completely anti-synchronized (second and fourth states) so that complete synchronization (CS) and anti-synchronization (AS) coexist in the synchronization process of the two hyperchaotic systems (1) and (2).

A simple calculation yields the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= -a e_1 + a(y_2 - x_2) + y_2 y_3 - x_2 x_3 + u_1 \\
\dot{e}_2 &= b e_2 + b(y_1 + x_1) - (y_1 y_3 + x_1 x_3) + u_2 \\
\dot{e}_3 &= -c e_3 - \varepsilon(y_4 - x_4) + y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= -d e_4 + f(y_3 + x_3) + y_1 y_2 + x_1 x_2 + u_4
\end{aligned} \tag{4}$$

We consider the nonlinear controller defined by

$$\begin{aligned} u_1 &= -a(y_2 - x_2) - y_2 y_3 + x_2 x_3 \\ u_2 &= -2be_2 - b(y_1 + x_1) + y_1 y_3 + x_1 x_3 \\ u_3 &= \varepsilon(y_4 - x_4) - y_1 y_2 + x_1 x_2 \\ u_4 &= -f(y_3 + x_3) - (y_1 y_2 + x_1 x_2) \end{aligned} \tag{5}$$

Substitution of (5) into (4) yields the linear error dynamics

$$\dot{e}_1 = -ae_1, \ \dot{e}_2 = -be_2, \ \dot{e}_3 = -ce_3, \ \dot{e}_4 = -de_4 \tag{6}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{7}$$

Differentiating (7) along the trajectories of the system (6), we get

$$\dot{V}(e) = -ae_1^2 - be_2^2 - ce_3^2 - de_4^2$$

which is a negative definite function on $\boldsymbol{R}^4$ since a, b, c and d are positive constants.

Thus, by Lyapunov stability theory [24], the error dynamics (6) is globally exponentially stable. Hence, we obtain the following result.

Theorem 1. The identical hyperchaotic Qi systems (1) and (2) are globally and exponentially hybrid synchronized with the active nonlinear controller (5). ■

Numerical Simulations

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (1) and (2) with the nonlinear controller (5).

The parameters of the identical hyperchaotic Qi systems (1) and (2) are selected as

$$a = 50, \ b = 24, \ c = 13, \ d = 8, \ \ \varepsilon = 33, \ \ f = 30$$

so that the systems (1) and (2) exhibit hyperchaotic behaviour.

The initial values for the master system (1) are taken as

$$x_1(0) = 10, \ \ x_2(0) = 15, \ \ x_3(0) = 20, \ \ x_4(0) = 25$$

and the initial values for the slave system (2) are taken as

$$y_1(0) = 30, \ \ y_2(0) = 25, \ \ y_3(0) = 10, \ \ y_4(0) = 8$$

Figure 2 exhibits the hybrid synchronization of the hyperchaotic systems (1) and (2).

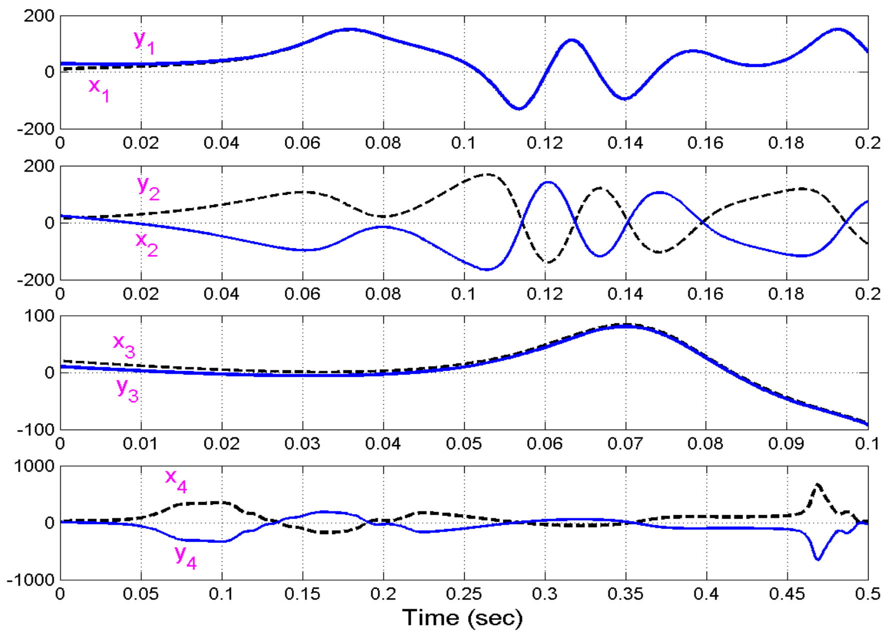

Figure 2. Hybrid Synchronization of the Identical Qi Systems

3. HYBRID SYNCHRONIZATION OF IDENTICAL JIA SYSTEMS

In this section, we consider the hybrid synchronization of identical hyperchaotic Jia systems [23]. Thus, we consider the master system as the hyperchaotic Jia dynamics described by

$$\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\
\dot{x}_2 &= -x_1 x_3 + \beta x_1 - x_2 \\
\dot{x}_3 &= x_1 x_2 - \gamma x_3 \\
\dot{x}_4 &= -x_1 x_3 + \delta x_4
\end{aligned} \tag{8}$$

where $x_i \ (i = 1,2,3,4)$ are the state variables and $\alpha, \beta, \gamma, \delta$ are positive constants.

We consider the hyperchaotic Jia dynamics also as the *slave* system, which is described by

$$\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= -y_1 y_3 + \beta y_1 - y_2 + u_2 \\
\dot{y}_3 &= y_1 y_2 - \gamma y_3 + u_3 \\
\dot{y}_4 &= -y_1 y_3 + \delta y_4 + u_4
\end{aligned} \tag{9}$$

where $y_i \ (i = 1,2,3,4)$ are the state variables and $u_i \ (i = 1,2,3,4)$ are the active controls.

When $\alpha = 10, \ \beta = 28, \ \gamma = 8/3$ and $\delta = 1.3$, the Jia system (8) is hyperchaotic (see Figure 3).

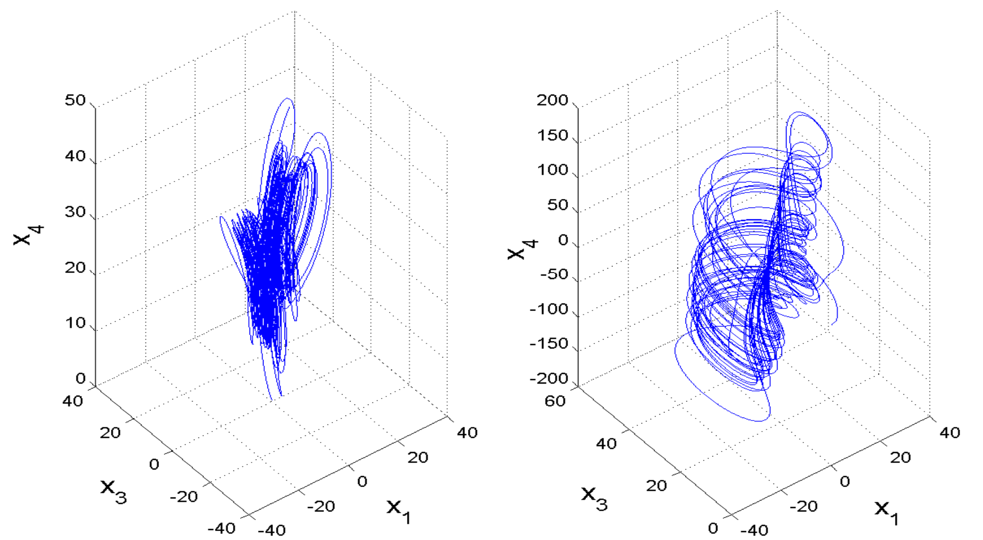

Figure 3. State Orbits of the Hyperchaotic Jia System (8)

For the hybrid synchronization of the identical hyperchaotic Jia systems (8) and (9), the synchronization errors are defined as

$$e_1 = y_1 - x_1, \;\; e_2 = y_2 + x_2, \; e_3 = y_3 - x_3, \;\; e_4 = y_4 + x_4 \tag{10}$$

From the error equations (10), it is clear that one part of the two hyperchaotic systems is completely synchronized (first and third states), while the other part is completely anti-synchronized (second and fourth states) so that complete synchronization (CS) and anti-synchronization (AS) coexist in the synchronization process of the two hyperchaotic systems (8) and (9).

A simple calculation yields the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 - 2\alpha x_2 - 2x_4 + u_1 \\
\dot{e}_2 &= -e_2 + \beta e_1 + 2\beta x_1 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\gamma e_3 + y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= \delta e_4 - y_1 y_3 - x_1 x_3 + u_4
\end{aligned} \tag{11}$$

We consider the nonlinear controller defined by

$$\begin{aligned}
u_1 &= -\alpha e_2 - e_4 + 2x_4 + 2\alpha x_2 \\
u_2 &= -\beta e_1 - 2\beta x_1 + y_1 y_3 + x_1 x_3 \\
u_3 &= -y_1 y_2 + x_1 x_2 \\
u_4 &= -(\delta + 1) e_4 + y_1 y_3 + x_1 x_3
\end{aligned} \tag{12}$$

Substitution of (12) into (11) yields the linear error dynamics

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 \\ \dot{e}_2 &= -\beta e_2 \\ \dot{e}_3 &= -\gamma e_3 \\ \dot{e}_4 &= -\delta e_4 \end{aligned} \tag{13}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{14}$$

Differentiating (14) along the trajectories of the system (13), we get

$$\dot{V}(e) = -\alpha e_1^2 - \beta e_2^2 - \gamma e_3^2 - \delta e_4^2$$

which is a negative definite function on $\boldsymbol{R}^4$ since α, β, γ and δ are positive constants.

Thus, by Lyapunov stability theory [24], the error dynamics (13) is globally exponentially stable. Hence, we obtain the following result.

Theorem 2. The identical hyperchaotic Jia systems (8) and (9) are globally and exponentially hybrid synchronized with the active nonlinear controller (12). ■

Numerical Simulations

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (8) and (9) with the nonlinear controller (12).

The parameters of the identical hyperchaotic Jia systems (8) and (9) are selected as

$$\alpha = 10, \ \beta = 28, \ \gamma = 8/3, \ \delta = 1.3$$

so that the systems (8) and (9) exhibit hyperchaotic behaviour.

The initial values for the master system (8) are taken as

$$x_1(0) = 8, \ \ x_2(0) = 20, \ \ x_3(0) = 10, \ \ x_4(0) = 15$$

and the initial values for the slave system (9) are taken as

$$y_1(0) = 16, \ \ y_2(0) = 10, \ \ y_3(0) = 20, \ \ y_4(0) = 22$$

Figure 4 exhibits the hybrid synchronization of the hyperchaotic systems (8) and (9).

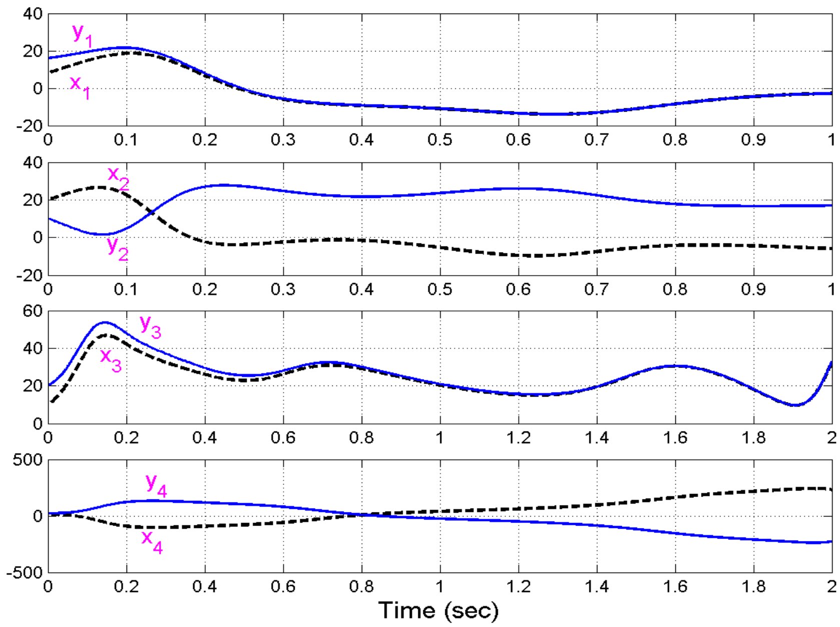

Figure 4. Hybrid Synchronization of the Identical Jia Systems

4. Hybrid Synchronization of Qi and Jia Systems

In this section, we consider the hybrid synchronization of non-identical hyperchaotic Qi [22] and Jia [23] systems.

Thus, we consider the master system as the hyperchaotic Qi dynamics described by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\
\dot{x}_3 &= -c x_3 - \varepsilon x_4 + x_1 x_2 \\
\dot{x}_4 &= -d x_4 + f x_3 + x_1 x_2
\end{aligned} \tag{15}$$

where $x_i \ (i = 1, 2, 3, 4)$ are the state variables and $a, b, c, d, \varepsilon, f$ are positive constants.

We consider the hyperchaotic Jia as the *slave* system, which is described by

$$\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= -y_1 y_3 + \beta y_1 - y_2 + u_2 \\
\dot{y}_3 &= y_1 y_2 - \gamma y_3 + u_3 \\
\dot{y}_4 &= -y_1 y_3 + \delta y_4 + u_4
\end{aligned} \tag{16}$$

where $y_i \ (i = 1, 2, 3, 4)$ are the state variables, $\alpha, \beta, \gamma, \delta$ are positive constants and $u_i \ (i = 1, 2, 3, 4)$ are the active controls.

For the hybrid synchronization of the hyperchaotic systems (15) and (16), the synchronization errors are defined as

$$e_1 = y_1 - x_1, \;\; e_2 = y_2 + x_2, \; e_3 = y_3 - x_3, \;\; e_4 = y_4 + x_4 \tag{17}$$

From the error equations (17), it is clear that one part of the two hyperchaotic systems is completely synchronized (first and third states), while the other part is completely anti-synchronized (second and fourth states) so that complete synchronization (CS) and anti-synchronization (AS) coexist in the synchronization process of the two hyperchaotic systems (15) and (16).

A simple calculation yields the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + (a-\alpha)x_1 - (a+\alpha)x_2 + y_4 - x_2 x_3 + u_1 \\
\dot{e}_2 &= \beta e_1 - e_2 + (\beta + b)x_1 + (b+1)x_2 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\gamma e_3 + (c-\gamma)x_3 + \varepsilon x_4 - x_1 x_2 + y_1 y_2 + u_3 \\
\dot{e}_4 &= \delta e_4 - (d+\delta)x_4 + f x_3 + x_1 x_2 - y_1 y_3 + u_4
\end{aligned} \tag{18}$$

We consider the nonlinear controller defined by

$$\begin{aligned}
u_1 &= -\alpha e_2 + (\alpha - a)x_1 + (a+\alpha)x_2 - y_4 + x_2 x_3 \\
u_2 &= -\beta e_1 - (\beta + b)x_1 - (b+1)x_2 + y_1 y_3 + x_1 x_3 \\
u_3 &= (\gamma - c)x_3 - \varepsilon x_4 + x_1 x_2 - y_1 y_2 \\
u_4 &= -(\delta + 1)e_4 + (d+\delta)x_4 - f x_3 - x_1 x_2 + y_1 y_3
\end{aligned} \tag{19}$$

Substitution of (19) into (18) yields the linear error dynamics

$$\dot{e}_1 = -ae_1, \; \dot{e}_2 = -be_2, \; \dot{e}_3 = -ce_3, \; \dot{e}_4 = -de_4 \tag{20}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{21}$$

Differentiating (21) along the trajectories of the system (20), we get

$$\dot{V}(e) = -ae_1^2 - be_2^2 - ce_3^2 - de_4^2$$

which is a negative definite function on $\boldsymbol{R}^4$ since a, b, c and d are positive constants.

Thus, by Lyapunov stability theory [24], the error dynamics (20) is globally exponentially stable. Hence, we obtain the following result.

Theorem 3. The non-identical hyperchaotic Qi system (15) and hyperchaotic Jia system (16) are globally and exponentially hybrid synchronized with the active nonlinear controller (19). ■

Numerical Simulations

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (15) and (16) with the nonlinear controller (19).

The initial values for the master system (15) are taken as

$$x_1(0) = 20, \quad x_2(0) = 25, \quad x_3(0) = 10, \quad x_4(0) = 15$$

and the initial values for the slave system (16) are taken as

$$y_1(0) = 10, \quad y_2(0) = 12, \quad y_3(0) = 30, \quad y_4(0) = 26$$

The parameters of the master system (15) and slave system (16) are selected as

$$a = 50, b = 24, c = 13, d = 8, \ \varepsilon = 33, \ f = 30, \alpha = 10, \ \beta = 28, \ \gamma = 8/3, \ \delta = 1.3$$

so that the systems (15) and (16) undergo hyperchaotic behaviour.

Figure 5 exhibits the hybrid synchronization of the hyperchaotic systems (15) and (16).

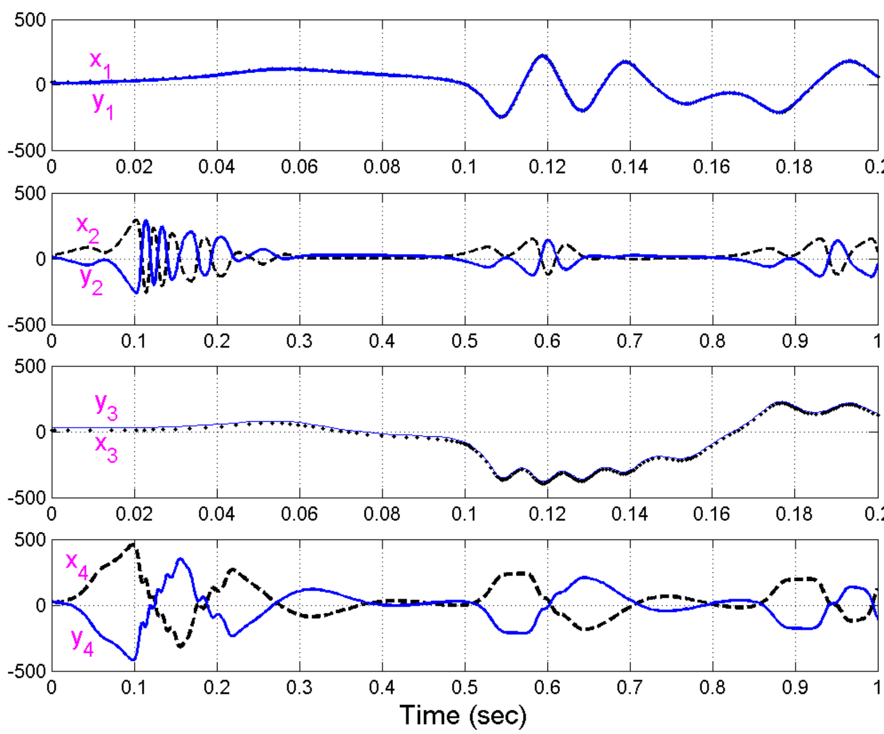

Figure 5. Hybrid Synchronization of Hyperchaotic Qi and Jia Systems

3. CONCLUSIONS

In this paper, nonlinear control method based on Lyapunov stability theory has been deployed to globally and exponentially hybrid synchronize the following hyperchaotic systems:

(A) Two identical hyperchaotic Qi systems (2008)

(B) Two identical hyperchaotic Jia systems (2007)

(C) Non-identical hyperchaotic Qi and Jia systems.

The stability results were established using Lyapunov stability theory. Numerical simulations have been given to demonstrate the effectiveness of the proposed hybrid synchronization schemes. Since Lyapunov exponents are not required for these calculations, the proposed nonlinear control method is effective and convenient to achieve hybrid synchronization of the hyperchaotic systems as mentioned in the three cases, (A)-(C).

REFERENCES

[1] Alligood, K.T., Sauer, T. & Yorke, J.A. (1997) *Chaos: An Introduction to Dynamical Systems*, Springer, New York.

[2] Pecora, L.M. & Carroll, T.L. (1990) "Synchronization in chaotic systems", *Phys. Rev. Lett.*, Vol. 64, pp 821-824.

[3] Lakshmanan, M. & Murali, K. (1996) *Nonlinear Oscillators: Controlling and Synchronization*, World Scientific, Singapore.

[4] Han, S.K., Kerrer, C. & Kuramoto, Y. (1995) "Dephasing and burstling in coupled neural oscillators", *Phys. Rev. Lett.,* Vol. 75, pp 3190-3193.

[5] Blasius, B., Huppert, A. & Stone, L. (1999) "Complex dynamics and phase synchronization in spatially extended ecological system", *Nature*, Vol. 399, pp 354-359.

[6] Feki, M. (2003) "An adaptive chaos synchronization scheme applied to secure communication", *Chaos, Solitons and Fractals*, Vol. 18, pp 141-148.

[7] Murali, K. & Lakshmanan, M. (1998) "Secure communication using a compound signal from generalized synchronizable chaotic systems", *Phys. Rev. Lett. A*, Vol. 241, pp 303-310.

[8] Yang, T. & Chua, L.O. (1999) "Control of chaos using sampled-data feedback control", Internat. J. Bifurcat. Chaos, Vol. 9, pp 215-219.

[9] Ott, E., Grebogi, C. & Yorke, J.A. (1990) "Controlling chaos", Phys. Rev. Lett., Vol. 64, pp 1196-1199.

[10] Park, J.H. & Kwon, O.M. (2003) "A novel criterion for delayed feedback control of time-delay chaotic systems", *Chaos, Solitons and Fractals*, Vol. 17, pp 709-716.

[11] Yu, Y.G. & Zhang, S.C. (2006) "Adaptive backstepping synchronization of uncertain chaotic systems", *Chaos, Solitons and Fractals*, Vol. 27, pp 1369-1375.

[12] Liao, T.L. & Tsai, S.H. (2000) "Adaptive synchronization of chaotic systems and its applications to secure communications", *Chaos, Solitons and Fractals*, Vol. 11, pp 1387-1396.

[13] Konishi, K.., Hirai, M. & Kokame, H. (1998) "Sliding mode control for a class of chaotic systems", *Phys. Lett. A*, Vol. 245, pp 511-517.

[14] Ge, Z.M. & Chen, C.C. (2004) "Phase synchronization of coupled chaotic multiple time scales systems", *Chaos, Solitons and Fractals*, Vol. 20, pp 639-647.

[15] Wang, Y.W. & Guan, Z.H. (2006) "Generalized synchronization of continuous chaotic systems", *Chaos, Solitons and Fractals*, Vol. 27, pp 97-101.

[16] Zhang, X. & Zhu, H. (2008) "Anti-synchronization of two different hyperchaotic systems via active and adaptive control", *Internat. J. Nonlinear Science*, Vol. 6, pp 216-223.

[17] Chiang, T., Lin, J., Liao, T. & Yan, J. (2008) "Anti-synchronization of uncertain unified chaotic systems with dead-zone nonlinearity", Nonlinear Analysis, Vol. 68, pp 2629-2637.

[18] Qiang, J. (2007) "Projective synchronization of a new hyperchaotic Lorenz system", *Phys. Lett. A,* Vol. 370, pp 40-45.

[19] Jian-Ping, Y. & Chang-Pin, L. (2006) "Generalized projective synchronization for the chaotic Lorenz system and the chaotic Chen system", *J. Shanghai Univ*., Vol. 10, pp 299-304.

[20] Li, R.H., Xu, W. & Li, S. (2007) "Adaptive generalized projective synchronization in different chaotic systems based on parameter identification", *Phys. Lett. A*, Vol. 367, pp 199-206.

[21] Li, R-H. (2008) "A special full-state hybrid synchronization in symmetrical chaotic systems", *Applied Math. Comput.,* Vol. 200, pp 321-319.

[22] Qi, G., Wyk, M.A., Wyk, B.J. & Chen, G. (2008) "On a new hyperchaotic system", *Phys. Lett. A*, Vol. 372, pp 124-136.

[23] Qiang Jia (2007) "Hyperchaos generated from the Lorenz system and its control", *Phys. Lett. A,* Vol. 366, pp 217-222.

[24] Hahn, W. (1967) *The Stability of Motion*, Springer, New York.

[25] Kamaraju, M., Kishore, K.L. & Tilak, A.V.N. (2010) "Power optimized programmable embedded controller", *Internat. Journal of Computer Networks & Commun.*, Vol. 2, pp. 97-107.

[26] Yuste, A.J., Trivino, A., Trujillo, F.D. & Casilari, E. (2010) "Using fuzzy logic in hybrid multihop wireless networks", *Internat. J. Wireless and Mobile Networks*, Vol. 2, pp. 96-108.

[27] Obermeier, S. & Böttcher, S. (2010) "A DBMS for mobile transactions using bi-state-termination", Internat. J. Database Management Systems, Vol. 2, pp. 141-159.

[28] Cannady, J. (2010) "Dynamic neural networks in the detection of distributed attacks in mobile ad-hoc networks", *Internat. Journal of Network Security & Its Applications*, Vol. 2, pp. 1-7.