ROBUST SECOND ORDER SLIDING MODE CONTROL FOR A QUADROTOR CONSIDERING MOTOR DYNAMICS

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ABSTRACT

In this paper, a robust second order sliding mode control (SMC) for controlling a quadrotor with uncertain parameters presented based on high order sliding mode control (HOSMC). A controller based on the HOSMC technique is designed for trajectory tracking of a quadrotor helicopter with considering motor dynamics. The main subsystems of quadrotor (i.e. position and attitude) stabilized using HOSMC method. The performance and effectiveness of the proposed controller are tested in a simulation study taking into account external disturbances with consider to motor dynamics. Simulation results show that the proposed controller eliminates the disturbance effect on the position and attitude subsystems efficiency that can be used in real time applications.

KEYWORDS

Quadrotor, High order sliding mode, Motor dynamics

1. INTRODUCTION

Nowadays, the use and development of unmanned aerial vehicles (UAV) in the aerospace industry have become a growing issue. Research groups around the world are attracted to these vehicles due to their numerous applications such as surveillance, inspection, search and rescue, among others. Moreover, the field of UAV includes many engineering challenges in the areas of electrical, mechanical and control engineering.

The quadrotor helicopter is classified as a rotary wing vertical take-off and landing (VTOL) aircraft equipped with four rotors. This rotorcraft has many advantages over conventional helicopters that can be cited to simple design, lower risk of damaging, ease of both construction, maneuverability, motion control and cost [1-2].

In order to meet the requirements for achieving autonomous flight, several control methods have been applied to the quadrotor such as input-output linearization [3], backstepping [4-5], PD sliding mode [6], adaptive control [7], PID controller augmented with feedback [8], integral predictive/nonlinear H_∞ [9], among others.
However, most of these methods require an exact knowledge of the system dynamics. An automatic control of UAV must be able to withstand the parametric uncertainties, unmodeled dynamics and handle the effect of wind and turbulence of motors. On the other hand, sliding

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mode controller ensures robustness only with respect to the matched perturbation [10], while the quadrotor dynamics are affected by both matched and unmatched perturbations [11].

To overcome the mentioned problem, a dynamic sliding mode feedback controller with wind parameter estimation has been designed in [12] where it is clear that this controller is not robust in the face of wind disturbances that affect the forces dynamic model of the displacement (x,y). Moreover, to overcome this problem, a HOSMC combined with robust differentiator has been proposed [13] where, to decouple the control inputs, two integrators were added to the control scheme increasing the system relative degree and the order of the designed robust differentiators. The robustness of this designed controller is guaranteed, but the transient of the system is quite long affecting the maneuverability of quadrotor.

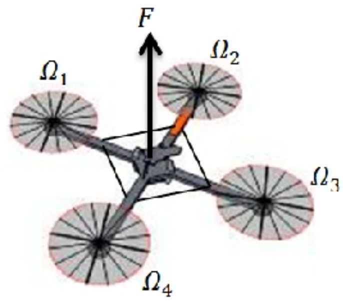
To overcome these drawbacks, in this paper, considering the nonlinear model of the UAV quadrotor and motor dynamics, the Higher Order Sliding Mode Control (HOSMC) technique [14-15] is used to design a robust flight controller capable to track and control the absolute position and attitude of the rotorcraft. The use of high order mode control techniques has led to satisfactory results in both the attitude stabilization [16] and tracking control [17].

First, the HOSMC technique is used to design a smooth altitude control law that allows the design of HOSM control algorithms for the longitudinal, latitudinal and heading motions control loops independently. Then, the super-twisting algorithm is implemented to make this manifold attractive bringing robustness to the closed-loop system while avoiding the chattering phenomenon [18]. The stability and finite time convergence characteristics of the proposed controller are studied using Lyapunov functions [15].

The paper is organized as follows: in Section (2), a dynamic nonlinear model considering motor dynamics is described for a miniature quadrotor helicopter. Based on this nonlinear model, a controller using HOSMC techniques is designed to control the position and attitude subsystems in Section (3). The designed controllers facilitate the sliding manifold design and bring robustness of the closed-loop system. Simulation results showing better performance of the proposed controller are presented in Section (4), and some conclusions close the paper.

2. Dynamic Modelling of a Quadrotor

The quadrotor configuration has four rotors which generate the propeller forces F_i (i=1, 2, 3, 4). Each rotor consists of an outer-rotor BLDC motor and a fixed-pitch propeller. This aircraft is constituted by two rotors (1,3) which rotate clockwise, and two (2, 4) rotating counter clockwise. In order to increase the altitude of the aircraft, it is necessary to increase the rotor speeds at the same rates. Forward motion is accomplished by increasing the speed of the rear rotor (3) while simultaneously reducing the same value for the forward rotor (1). Backward, leftward and rightward motion can be accomplished similarly, whereas yaw motion can be performed by speeding up or slowing down the clockwise rotors depending on the desired angle direction as shown in figure1.



$\Omega_1 = \Omega_3 = \Omega_2 = \Omega_4$	Move to up (U_1)
$(\Omega_1 = \Omega_3) > \Omega_2 < \Omega_4$	Move to right (U_2)
$\Omega_1 > \Omega_3 < (\Omega_2 = \Omega_4)$	Move to front (U_3)
$(\Omega_1 = \Omega_3) < (\Omega_2 = \Omega_4)$	Clockwise rotation (U_4)

Figure 1. Description of the quadrotor motion.

The quadrotor is an underactuated and highly nonlinear dynamic system, thus an appropriate model ideally includes the motors dynamics and gyroscopic effects resulting from both the rigid body rotation in space and the four propulsion groups of rotation [17]. The quadrotor model (position and attitude dynamics) obtained here is given in [19] and [20] by:

Translational and rotational motion equations are:

(1)

(2)

Where

(3)

U_1, U_2, $U_3$and U_4 are the control inputs of the system which are written according to the angular velocities of the four rotors as follows:

$$\begin{cases} U_1 = b\left(\Omega_1{}^2 + \Omega_2{}^2 + \Omega_3{}^2 + \Omega_4{}^2\right) \\ U_2 = bl\left(-\Omega_2{}^2 + \Omega_4{}^2\right) \\ U_3 = bl\left(-\Omega_1{}^2 + \Omega_3{}^2\right) \\ U_4 = d(-\Omega_1{}^2 + \Omega_2{}^2 - \Omega_3{}^2 + \Omega_4{}^2) \\ \Omega = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \end{cases} \tag{4}$$

Also, motors nonlinear equations are [21]:

$$\begin{cases} \dot{\Omega}_1 = -\dfrac{K_E K_M}{R\, J_{TP}} \Omega_1 - \dfrac{d}{J_{TP}} {\Omega_1}^2 + \dfrac{K_M}{R\, J_{TP}} v_1 \\ \dot{\Omega}_2 = -\dfrac{K_E K_M}{R\, J_{TP}} \Omega_2 - \dfrac{d}{J_{TP}} {\Omega_2}^2 + \dfrac{K_M}{R\, J_{TP}} v_2 \\ \dot{\Omega}_3 = -\dfrac{K_E K_M}{R\, J_{TP}} \Omega_3 - \dfrac{d}{J_{TP}} {\Omega_3}^2 + \dfrac{K_M}{R\, J_{TP}} v_3 \\ \dot{\Omega}_4 = -\dfrac{K_E K_M}{R\, J_{TP}} \Omega_4 - \dfrac{d}{J_{TP}} {\Omega_4}^2 + \dfrac{K_M}{R\, J_{TP}} v_4 \end{cases} \tag{5}$$

Where physical parameters of a quadrotorforsix DOF equations and motors are given in an appendix in Table1.Because of fast dynamic of motors, many researchers use models based on approximation in a model of quadrature [1], [2], [19] and [21], thus there are few literatures that consider the dynamics of the motors for controlling quadrotor [22-23].

Using of motor dynamics, adds four new states to main states (i.e. dynamics of position and attitude, linear and angular velocities) that it causes to control quadrotor difficulty. Therefore, it should present a new approach to control a quadrotor. In this paper, a different approach has been used to control quadrotor that is based on derived desired output speeds with the inverse relation of motors.The used algorithm in control of a typical quadrotor considering motor dynamics is presented in the flowchart in figure 2.

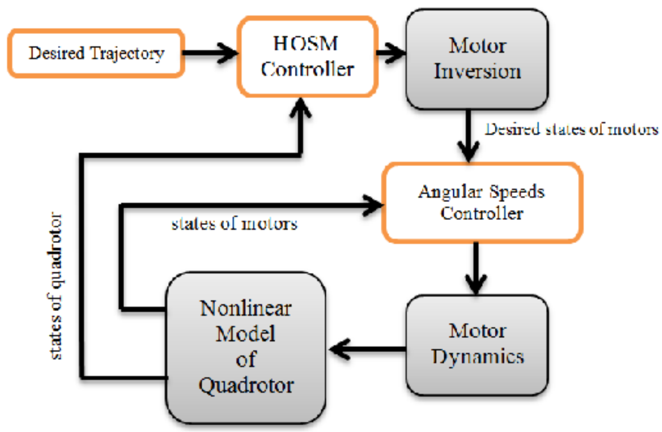


Figure 2. Proposed schematic for full control of a quadrotor via motor dynamics

The HOSM controller will be designed in the next section. In motor inversion block, the angular speeds of the four rotors can be written according to the control inputs as follows:

$$\begin{cases} {\Omega_1}^2 = \dfrac{1}{4b} F - \dfrac{1}{2bl} \tau_\theta - \dfrac{1}{4d} \tau_\psi \\ {\Omega_2}^2 = \dfrac{1}{4b} F - \dfrac{1}{2bl} \tau_\phi + \dfrac{1}{4d} \tau_\psi \\ {\Omega_3}^2 = \dfrac{1}{4b} F + \dfrac{1}{2bl} \tau_\theta - \dfrac{1}{4d} \tau_\psi \\ {\Omega_4}^2 = \dfrac{1}{4b} F + \dfrac{1}{2bl} \tau_\phi + \dfrac{1}{4d} \tau_\psi \end{cases} \tag{6}$$

Where F, τ_ϕ, τ_θ , τ_ψ are control inputs for controlling z axis, □angle, θ angle and ψ angle, respectively. These angular speeds have been considered as desired angular speeds. In other side, four angular speeds are considered as actual angular speeds. It will be useful that we can control the error between them. It can be done with a typical control method like PID controller or other appropriate control algorithm either linear or nonlinear controllers.

3. Control Design

The main objective of an UAV controller is to ensure the asymptotic convergence of the variable state vector $[x, y, z, \psi]^T$ to the reference trajectory $[x^r, y^r, z^r, \psi^r]^T$. This can be done by using a sliding mode control approaches. The mathematical model (1), (2) developed in Section (2) is used to describe the system in the state space form [11] and [20]. In this paper, voltages of motors are considered as inputs of quadrotor. It is worthwhile to note that the translations depend on the angles, therefore the nonlinear model (1), (2) can be rearranged in a state space form as follows:

$$
\begin{aligned}
S_1: &\begin{cases} \dot{x}_7 = x_8 \\ \dot{x}_8 = -g + (\mathrm{C}_{x_3}\,\mathrm{C}_{x_1})\dfrac{b\left({\Omega_1}^2 + {\Omega_2}^2 + {\Omega_3}^2 + {\Omega_4}^2\right)}{m} + W_z \end{cases} \\
S_2: &\begin{cases} \dot{x}_9 = x_{10} \\ \dot{x}_{10} = (\mathrm{S}_{x_1}\,\mathrm{S}_{x_5} + \mathrm{C}_{x_1}\,\mathrm{S}_{x_3}\,\mathrm{C}_{x_5})\dfrac{b\left({\Omega_1}^2 + {\Omega_2}^2 + {\Omega_3}^2 + {\Omega_4}^2\right)}{m} + W_x \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{I_z - I_x}{I_y}pr - \dfrac{J_{TP}}{I_y}p\Omega + \dfrac{bl\left(-{\Omega_1}^2 + {\Omega_3}^2\right)}{I_y} + W_{\dot{\theta}} \end{cases} \\
S_3: &\begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = (-\mathrm{C}_\psi\,\mathrm{S}_\phi + \mathrm{S}_\psi\,\mathrm{S}_\theta\,\mathrm{C}_\phi)\dfrac{b\left({\Omega_1}^2 + {\Omega_2}^2 + {\Omega_3}^2 + {\Omega_4}^2\right)}{m} + W_y \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{I_y - I_z}{I_x}qr - \dfrac{J_{TP}}{I_x}q\Omega + \dfrac{bl\left(-{\Omega_2}^2 + {\Omega_4}^2\right)}{I_x} + W_{\dot{\phi}} \end{cases} \\
S_4: &\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = \dfrac{I_x - I_y}{I_z}pq + \dfrac{d(-{\Omega_1}^2 + {\Omega_2}^2 - {\Omega_3}^2 + {\Omega_4}^2)}{I_z} + W_{\dot{\psi}} \end{cases}
\end{aligned}
\qquad (7)
$$

$x_7 = z$, $x_8 = \dot{z}$, $x_9 = x$, $x_{10} = \dot{x}$
$x_{11} = y$, $x_{12} = \dot{y}$, $x_7 = z$, $x_8 = \dot{z}$
$x_1 = \phi$, $x_2 = \dot{\phi}$ (ϕ □ 0), $x_3 = \theta$, $x_4 = \dot{\theta}$(θ □ 0)
$x_5 = \psi$, $x_6 = \dot{\psi}$(ψ □ 0)

Where W_x, W_y, W_z and $W_{\dot{\phi}}, W_{\dot{\theta}}, W_{\dot{\psi}}$ are the resulting aerodynamic forces and moments expressed in the inertia reference frame, respectively [11]. Here, we suppose $\|W_z\|_\infty \le \overline{W_z}$, $\|W_x\|_\infty \le \overline{W_x}$, $\|W_y\|_\infty \le \overline{W_y}$, $\|W_{\dot{\psi}}\|_\infty \le \overline{W_{\dot{\psi}}}$, $\|W_{\dot{\phi}}\|_\infty \le \overline{W_{\dot{\phi}}}$, $\|W_{\dot{\theta}}\|_\infty \le \overline{W_{\dot{\theta}}}$ where $\overline{\mathrm{W_z}} > 0$, $\overline{\mathrm{W_x}} > 0$, $\overline{\mathrm{W_y}} > 0$, $\overline{\mathrm{W}_{□}} > 0$, $\overline{\mathrm{W}_{\dot{\theta}}} > 0$ and $\overline{\mathrm{W}_{\dot{\psi}}} > 0$. Moreover, it is a good idea that one considers motor dynamics in design of controller for real world applications.

$$S_5:\begin{cases}\dot{\Omega}_1 = -\frac{K_E K_M}{R\, J_{TP}}\Omega_1 - \frac{d}{J_{TP}}\Omega_1{}^2 + \frac{K_M}{R\, J_{TP}}v_1\\ \dot{\Omega}_2 = -\frac{K_E K_M}{R\, J_{TP}}\Omega_2 - \frac{d}{J_{TP}}\Omega_2{}^2 + \frac{K_M}{R\, J_{TP}}v_2\\ \dot{\Omega}_3 = -\frac{K_E K_M}{R\, J_{TP}}\Omega_3 - \frac{d}{J_{TP}}\Omega_3{}^2 + \frac{K_M}{R\, J_{TP}}v_3\\ \dot{\Omega}_4 = -\frac{K_E K_M}{R\, J_{TP}}\Omega_4 - \frac{d}{J_{TP}}\Omega_4{}^2 + \frac{K_M}{R\, J_{TP}}v_4\end{cases} \tag{8}$$

The problem of trajectory tracking is thus divided in the respective problem for five subsystems: altitudeS_1, longitudinal S_2, latitudinal S_3, heading control S_4 and angular speeds control S_5. It can be noted that aerodynamic moments$W_{\dot{\phi}}$,$W_{\dot{\theta}}$,$W_{\dot{\psi}}$ and aerodynamic force W_z are matched disturbances while the rest of the aerodynamic forces, W_x and W_y are unmatched ones. The control design for each subsystem will be carried out in the following subsections considering first that external disturbances resulting from the aerodynamic forces and moments are known and bounded. Then, a sliding mode controller will be designed to control dynamics of a quadrotor in presence of external disturbances.

The translational motion control is performed in two stages. In the first one, the helicopter height z, is controlled and the total thrust F, is the manipulated signal. In the second stage, the reference of pitch and roll angles ($\theta^r$and ϕ^r, respectively) are generated through the two virtual inputs U_x and U_y, computed to follow the desired $x - y$ movement [24]. Finally, the rotation controller is used to stabilize the quadrotor under near quasi-stationary conditions with control inputs τ_ϕ, τ_θ, τ_ψ.

3.1 Altitude Control (S_1)

Introducing the reference trajectory x_7^r for the altitude variable x_7 (*z* position), one can be defined the tracking error as:

$$e_z = x_7^r - x_7 \tag{9}$$

To stabilize the dynamics of e_z in Eqs. (7),(9), following the HOSM technique [15], the sliding surface S_z is chosen as:

$$S_z = x_8 - \dot{x}_7^r - k_1 e_z,\ \ k_1 > 0 \tag{10}$$

If disturbance is not exists in z position, the smooth control function F_{eq} can be obtained by $\dot{S}_z = 0$ using Eq. (7) as follows:

$$\dot{S}_z = -g + \frac{(C_{x_3}\, C_{x_1})}{m}F_{eq} - \ddot{x}_7^r - k_1(\dot{x}_7^r - x_8) = 0 \tag{11}$$

Where from Eq. (10), one can be obtained$x_8$as $x_8 = \dot{x}_7^r + k_1(x_7^r - x_7) + S_z$. Substituting x_8 in $\dot{S}_z = 0$, F_{eq} can be computed as :

$$F_{eq} = \frac{m}{(C_{x_3}\, C_{x_1})}\left(g + \ddot{x}_7^r - k_1{}^2(x_7^r - x_7) - k_1 S_z\right) \tag{12}$$

If there is disturbance in z position, it needs to increase the attractive area [25].

Note: in general, the entire control law is designed as $F = F_{eq} - v$where$v$ is the switching control part of the HOSMC and has been designed based on higher order sliding mode according to super twisting technique here [15].

Considering the mentioned note, altitude controller will be as follows:

$$F = F_{eq} - \frac{m}{(C_{x_3} C_{x_1})} (k_{1z}|S_z|^{\frac{1}{2}} sign\,(S_z) + k_{2z} \int |S_z|^{\frac{1}{3}}\, sign\,(S_z)) \tag{13}$$

The desired roll and pitch angles in terms of errors between actual and desired speeds are given separately by:

$$\begin{cases} \phi^r = arctg\left(\frac{U_y}{U_1}\right) \\ \theta^r = arctg\left(\frac{U_x}{U_1}\cos\phi\right) \end{cases} \tag{14}$$

3.2 Longitudinal Motion Control (S_2)

Introducing the tracking error for x position of the quadrotor as:

$$e_x = x_9^r - x_9 \tag{15}$$

To stabilize the dynamics of e_x in Eqs. (7), (15), similar to previous section, the sliding surface S_x is chosen as:

$$S_x = x_{10} - \dot{x}_9^r - k_2 e_x \quad , \quad k_2 > 0 \tag{16}$$

If disturbance is not exists in x position, the smooth control function $U_{x_{eq}}$ can be obtained by $\dot{S}_x = 0$ via Eq. (16) as follows:

$$\dot{S}_x = \tilde{u}_{x_{eq}} - \ddot{x}_9^r - k_2(\dot{x}_9^r - x_{10}) = 0 \tag{17}$$

Where from Eq. (16), one can be obtained x_{10}as $x_{10} = \dot{x}_9^r + k_2(x_9^r - x_9) + S_x$. Substituting x_{10} in $\dot{S}_x = 0$, $\tilde{u}_{x_{eq}}$ can be computed as :

$$\tilde{u}_{x_{eq}} = \left(\ddot{x}_9^r - {k_2}^2(x_9^r - x_9) - k_2 S_x\right) \tag{18}$$

If there is disturbance in x position, it needs to increase the attractive area. Considering the mentioned note, altitude controller will be as follows:

$$U_x = \tilde{u}_{x_{eq}} - (k_{1x}|S_x|^{\frac{1}{2}} sign\,(S_x) + k_{2x} \int |S_x|^{\frac{1}{3}}\, sign\,(S_x)) \tag{19}$$

From Eqs. (14)-(19), one can get θ^r as follows:

$$\theta^r = arctg\left(\frac{U_x}{U_1}\cos\phi\right) \tag{20}$$

3.3 Latitudinal Motion Control (S_3)

Introducing the tracking error for the y position of the quadrotor as:

$$e_y = x_{11}^r - x_{11} \tag{21}$$

To stabilize the dynamics of e_y in Eqs. (7), (21), similar to previous section, the sliding surface S_y is chosen as:

$$S_y = x_{12} - \dot{x}_{11}^r - k_3 e_y \ , \ k_3 > 0 \tag{22}$$

If disturbance is not exists in y position, the smooth control function $U_{y_{eq}}$ can be obtained by $\dot{S}_y = 0$ via Eq. (22) as follows:

$$\dot{S}_y = \tilde{u}_{y_{eq}} - \ddot{x}_{11}^r - k_3(\dot{x}_{11}^r - x_{12}) = 0 \tag{23}$$

Where from Eq. (22), one can be obtained x_{12} as $x_{12} = \dot{x}_{11}^r + k_3(x_{11}^r - x_{11}) + S_y$. Substituting x_{12} in $\dot{S}_y = 0$, $\tilde{u}_{y_{eq}}$ can be computed as:

$$\tilde{u}_{y_{eq}} = \left(\ddot{x}_{11}^r - {k_3}^2(x_{11}^r - x_{11}) - k_3 S_y\right) \tag{24}$$

If there is disturbance in y position, it needs to increase the attractive area. With consider to the mentioned note, altitude controller will be as follows:

$$U_y = \tilde{u}_{y_{eq}} - (\ k_{1y}|S_y|^{\frac{1}{2}} sign(S_y) + k_{2y} \int |S_y|^{\frac{1}{3}} sign(S_y)) \tag{25}$$

From Eqs. (25), (14), one can get ϕ^r as follows:

$$\phi^r = arctg\left(\frac{U_y}{U_1}\right) \tag{26}$$

The input signals of translational subsystem are x^r, y^r, z^r and its outputs are □^{r}and θ^{r}. These outputs are considered as desired inputs of attitude subsystem. Also, one can design the control inputs τ_ϕ and τ_θ to control ϕ angle and θ angle respectively where we have ($\phi \rightarrow \phi^r$ and $\theta \rightarrow \theta^r$). This process is shown in a flowchart in figure 3.

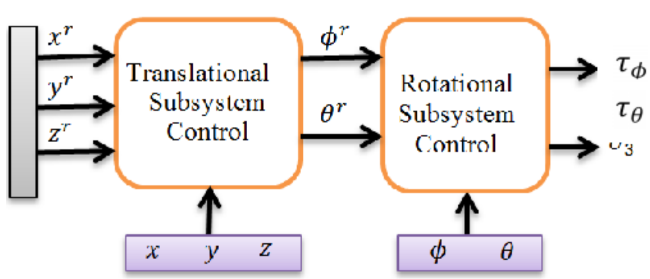


Figure 3. Proposed algorithm for control of a quadrotor.

3.4 Heading Control (S_4)

Introducing the tracking error for heading of the quadrotor ψ as:

$$e_\psi = x_5^r - x_5 \tag{27}$$

To stabilize the dynamics of e_ψ in Eq. (7), (27), similar to previous section, the sliding surface S_ψ is chosen as:

$$S_\psi = x_6 - \dot{x}_5^r - k_4 e_\psi \quad , \; k_4 > 0 \tag{28}$$

If disturbance is not exists in heading ψ, the smooth control function $\tilde{\tau}_{\psi_{eq}}$can be obtained by $\dot{S}_\psi = 0$ considering Eq. (28) as follows:

$$\dot{S}_\psi = \tilde{\tau}_{\psi_{eq}} - \ddot{x}_5^r - k_4(\dot{x}_5^r - x_6) = 0 \tag{29}$$

Where from Eq. (28), one can be obtained x_6 as $x_6 = \dot{x}_5^r + k_4(x_5^r - x_5) + S_\psi$. Substituting x_6 in $\dot{S}_\psi = 0$, $\tilde{\tau}_{\psi_{eq}}$ can be computed as:

$$\tilde{\tau}_{\psi_{eq}} = \left(\ddot{x}_5^r - {k_4}^2(x_5^r - x_5) - k_4 S_\psi\right) \tag{30}$$

If there is disturbance in heading ψ, it needs to increase the attractive area. Heading controller will be as follows:

$$\tau_\psi = \tilde{\tau}_{\psi_{eq}} - (k_{1\psi}|S_\psi|^{\frac{1}{2}} sign\left(S_\psi\right) + k_{2\psi} \int |S_\psi|^{\frac{1}{3}} sign\left(S_\psi\right)) \tag{31}$$

3.5 Angular Speeds Control (S_5)

In most references, because of the fast dynamics, motors of quadrotor have been approximated or neglected [20], [21]. A reason for it, modelling of robot in hover state while in the beginning and in running period, this dynamic effect on stability and performance of the robot for its initial take off. In this paper, the dynamics of motors are considered and a method for controlling quadrotor via angular speeds is presented. This method is based on error of angular speeds of a quadrotor that is shown in theflowchart in figure 4.

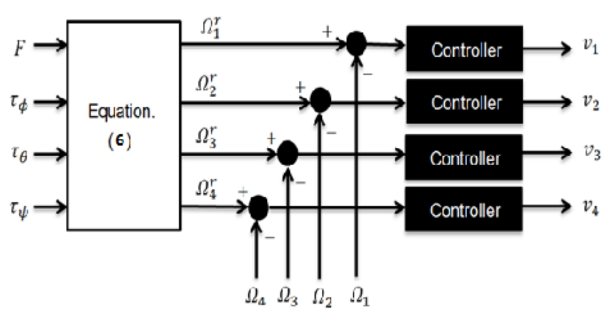


Figure 4. Flowchart for control of a quadrotor via angular speeds.

Control of angular speeds can be done using conventional and new approaches that a conventional proportional-integral-derivate (PID) is used here. Figure 5 shows a block diagram of full control of a quadrotor via angular speeds, that is used for simulation in this paper.

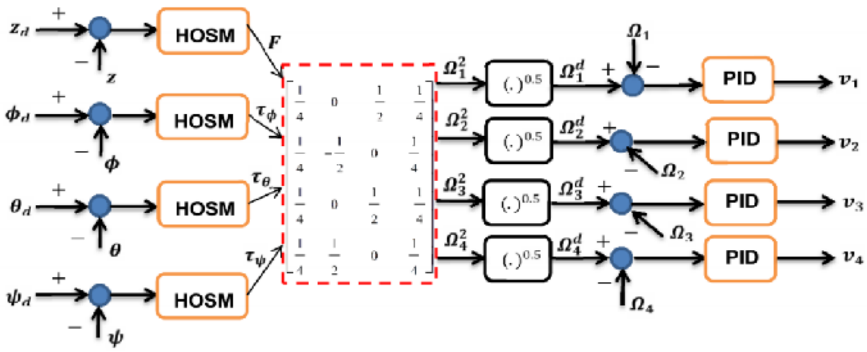


Figure 5. Implementation of proposed algorithm.

These control signals may play a role for input voltages to motors. After obtaining of the control laws, one requires to stability of the controller that is presented in following theorem. In other words, the control inputs F, U_x, U_y, τ_ψ should be designed to make S_z , S_x, S_y, S_ψ converge to the manifold $S_z = S_x = S_y = S_\psi = 0$, respectively.

Theorem: Under the designed high order sliding control laws (13) - (19) - (25) - (31), the statesof the under-actuated system of a quadrotor (7) will all asymptoticallyconverge to zero.

Prove. A Lyapunov function candidate can be selected as$V = \sum \frac{1}{2} S^T S$ that one can describe$V$ as [26]:

$$V = \frac{1}{2}(S_z^T S_z + S_x^T S_x + S_y^T S_y + S_\psi^T S_\psi) \tag{32}$$

Then, the time derivative of (32) is as:

$$\begin{aligned} \dot{V} &= (S_z^T \dot{S}_z + S_x^T \dot{S}_x + S_y^T \dot{S}_y + S_\psi^T \dot{S}_\psi) \\ &= (S_z^T\{-g + (C_{x_3} C_{x_1}) U_1 + W_z - \ddot{x}_7^r - k_1(\dot{x}_7^r - x_8)\} \\ &+ S_x^T\{(S_{x_1} S_{x_5} + C_{x_1} S_{x_3} C_{x_5}) U_x + W_x - \ddot{x}_9^r - k_2(\dot{x}_9^r - x_{10})\} \\ &+ S_y^T\{(-C_\psi S_\phi + S_\psi S_\theta C_\phi) U_y + W_y - \ddot{x}_{11}^r - k_3(\dot{x}_{11}^r - x_{12})\} \\ &+ S_\psi^T\left\{\frac{I_x - I_y}{I_z} pq + U_\psi + W_{\dot{\psi}} - \ddot{x}_5^r - k_4(\dot{x}_5^r - x_6)\right\}) \end{aligned} \tag{33}$$

With considering to $U = U_{eq} - v$ and substituting (12), (18), (24), and (30) in (33),$\dot{V}$ can be written to:

$$\begin{aligned} \dot{V} = (S_z^T \Big\{ -\frac{(C_{x_3} C_{x_1})}{m} v_z + W_z \Big\} &+ S_x^T \Big\{ -\frac{(S_{x_1} S_{x_5} + C_{x_1} S_{x_3} C_{x_5})}{m} v_x + W_x \Big\} \\ &+ S_y^T \Big\{ -\frac{(-C_\psi S_\phi + S_\psi S_\theta C_\phi)}{m} v_y + W_y \Big\} + S_\psi^T \Big\{ -\frac{1}{I_z} v_\psi + W_{\dot{\psi}} \Big\}) \end{aligned} \tag{34}$$

Let

$$v_z = \frac{m}{(C_{x_3} C_{x_1})} \left| M_z \left(|S_z|^{\frac{1}{2}} sign(S_z) + \int |S_z|^{\frac{1}{3}} sign(S_z) \right) + \eta_z S_z \right|,$$

$$v_x = \frac{m}{(S_{x_1} S_{x_5} + C_{x_1} S_{x_3} C_{x_5})} \left| M_x \left(|S_x|^{\frac{1}{2}} sign(S_x) + \int |S_x|^{\frac{1}{3}} sign(S_x) \right) + \eta_x S_x \right|,$$

$$v_y = \frac{m}{(-C_\psi\, S_\phi + S_\psi\, S_\theta\, C_\phi)} \left| M_y \left(\left|S_y\right|^{\frac{1}{2}} sign(S_y) + \int \left|S_y\right|^{\frac{1}{3}} sign(S_y) \right) + \eta_y S_y \right|, v_\psi = I_z \left| M_\psi \left(\left|S_\psi\right|^{\frac{1}{2}} sign(S_\psi) + \int \left|S_\psi\right|^{\frac{1}{3}} sign(S_\psi) \right) + \eta_\psi S_\psi \right|$$

Where

$M_Z = \bar{W}_z\|x_8\| + \rho_z > 0, M_x = \bar{W}_x\|x_{10}\| + \rho_x > 0, M_y = \bar{W}_y\|x_{12}\| + \rho_y > 0$, $M_\psi = \bar{W}_\psi\|x_6\| + \rho_\psi > 0$ and $\rho_z > 0$, $\rho_x > 0, \rho_y > 0$, $\rho_\psi > 0$ and $\eta_z > 0$, $\eta_x > 0$, $\eta_y > 0$, $\eta_\psi > 0$ are controller parameters. Then (34) becomes

$$\begin{aligned}
\dot{V} = \big(S_z^T\{-M_Z\, sign(S_z) - \eta_z S_z + W_z\} &+ S_x^T\{-M_x\, sign(S_x) - \eta_x S_x + W_x\} \\
&+ S_y^T\{-M_y\, sign(S_y) - \eta_y S_y + W_y\} \\
&+ S_\psi^T\{-M_\psi\, sign(S_\psi) - \eta_\psi S_\psi + W_{\dot\psi}\}\big) \\
&< -\eta_z S_z^T S_z - (M_z - \bar{W}_z\|x_8\|)\|S_z\|_1 - \eta_x S_x^T S_x - (M_x - \bar{W}_x\|x_{10}\|)\|S_x\|_1 \\
&- \eta_y S_y^T S_y - (M_y - \bar{W}_y\|x_{12}\|)\|S_y\|_1 - \eta_\psi S_\psi^T S_\psi - (M_\psi - \bar{W}_\psi\|x_6\|)\|S_\psi\|_1 \\
&< -\eta_z S_z^T S_z - \rho_z\|S_z\|_1 - \eta_x S_x^T S_x - \rho_x\|S_x\|_1 - \eta_y S_y^T S_y - \rho_y\|S_y\|_1 \\
&- \eta_\psi S_\psi^T S_\psi - \rho_\psi\|S_\psi\|_1 \leq 0
\end{aligned} \tag{35}$$

Therefore, under the designed control laws in (13), (19), (25) and (31), the under-actuated subsystem (7) will reach and thereafter stay on the manifolds $S_z = S_x = S_y = S_\psi = 0$ in finite time. Now, we will prove that on the sliding manifolds $S_z = S_x = S_y = S_\psi = 0$, the errors e_z, e_x, e_y, e_ψ will all asymptoticallyconverge to zero by picking up an appropriate set of k_i (i = 1, 2, 3, 4).

4. SIMULATION RESULTS

The proposed control strategy has been tested by simulations in order to check the performance attained for the path tracking problem. Simulations have been performed considering external disturbances and parametric uncertainties. The following vertical helix has been defined as the reference trajectory:

$$x^d(t) = \frac{t}{2}, y^d(t) = 0.1\sin(t/2), \quad z^d(t) = x(t), \quad \psi^d = 0$$

The considered initial conditions of the simulated quadrotor are (x, y, z) = (0, 0, 0.5) m and (ϕ, θ, ψ) = (0, 0, 0.5) rad. The values of the model parameters used for simulations are given in appendix in section A. An amount of ±20% in the uncertainty of the elements of the inertia matrix has been considered in the simulations. Figure6 to figure 9 present a perfect tracking of the reference trajectory when external disturbance originated by aerodynamic moments are considered. The results illustrate the robust performance provided by the controller in the case of parametric uncertainty in the inertia terms.

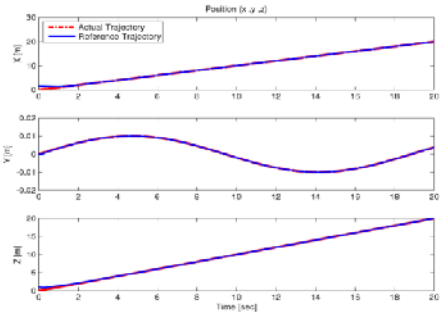


Figure 6. Position outputs of a quadrotor for a vertical helix.

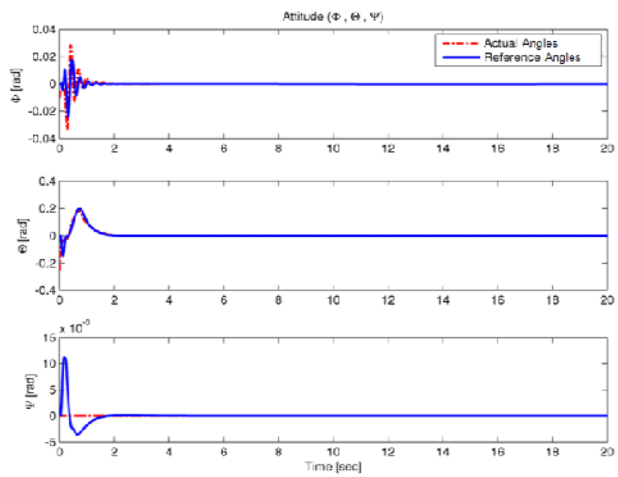


Figure 7. Angles outputs of a quadrotor for a vertical helix.

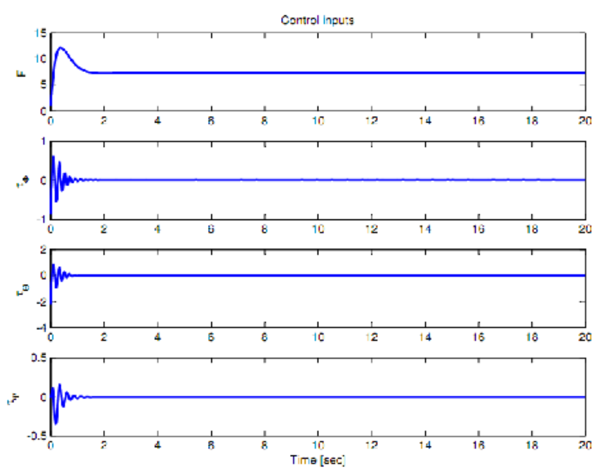


Figure 8. Higher order sliding mode control inputs (F, τ_{ϕ}, τ_{θ}, τ_{ψ})

Moreover, position and attitude errors in tracking of a quadrotor for vertical helix are shown in figure 9 and figure10, respectively. Using the HOSMC, a smooth reference tracking was performed, mainly, in the beginning of the tracking where the vehicle is far from the trajectory that causes an error. This is due to the different actual position and reference position in the beginning of motion. Then, the controller tries to decrease the error of tracking in next times during the simulation. Trajectory of a quadrotor in 3D space is shown in figure11.

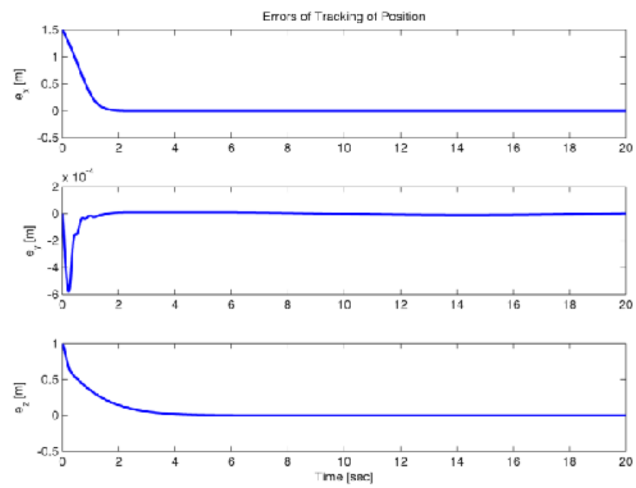


Figure 9. Position errors of a quadrotor for a vertical helix.

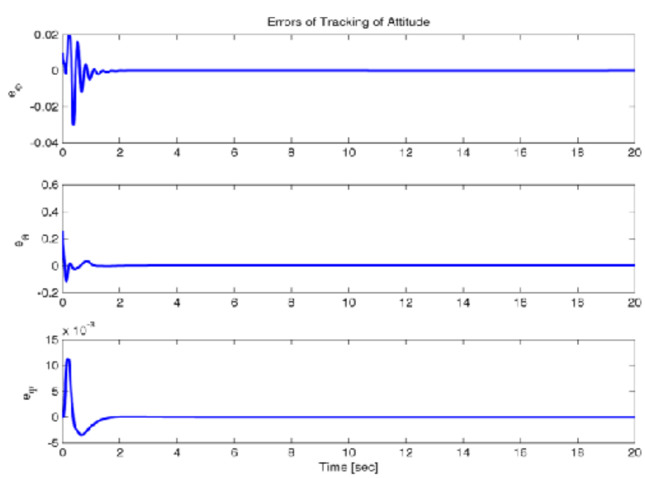


Figure 10. Attitude errors of a quadrotor for a vertical helix.

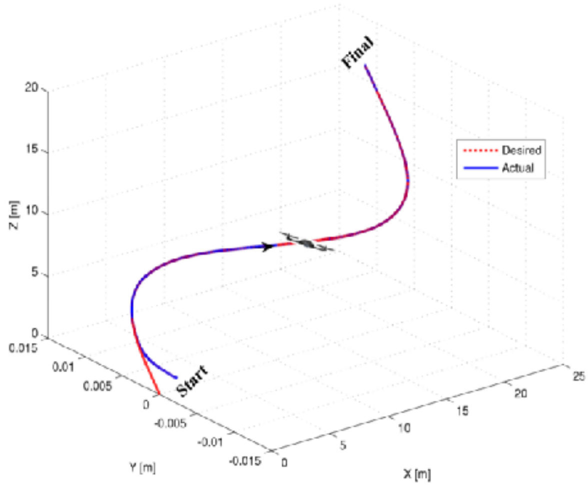


Figure11. Path tracking of quadrotor in 3D.

5. CONCLUSION

In this paper, a controller based on the high order sliding mode control technique has been used for the quadrotor helicopter. The HOSMC is used to add robustness to the closed-loop system while reducing the chattering phenomenon. Furthermore, the virtual control inputs and the wind parameter resulting from the aerodynamic forces have been handled via the higher order sliding mode. The system is divided into five subsystems where only one local control input is designed for each subsystem. This control law has led to satisfactory results in terms of trajectory tracking. Simulations show the good performance of the designed controller.

7. APPENDIX

7.1 Physical Specifications of Quadrotor

The considered parameters in simulation of a quadrotor in detail are given in following table 1:

Table 1: Physical parameters for simulation

Symbol	Quantity	value	unit
	Trust factor	13.33e-6	
	Drag factor	0.3e-6	
	Total mass of quadrotor	1.3	
	earth gravity	9.81	
	Body moment around *x* axis	8.1e-3	
	Body moment around *y* axis	8.1e-3	
	Body moment around *z* axis	14.2e-3	
	Distance from motors to center of quadrotor	0.27	
	rotational moment around propeller axis	3.33e-5	
	electrical constant of motor	8.3e-3	
	mechanical constant of motor	8.3e-3	
	Electrical resistance of motor	0.3	

7.2 Coefficients of HOSM controllers for Quadrotor

In this paper, the designed controllers based on high order sliding modes have coefficients that are given in Table 2:

Table 2: Physical parameters for simulation

Coefficient	Value
	10
	10
	1.3
	10
	1.5
	10
	5
	1.27
	3.5
	3.6
	4
	4.6

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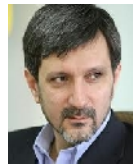

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