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ACTIVE CONTROLLER DESIGN FOR THE GENERALIZED PROJECTIVE SYNCHRONIZATION OF THREE-SCROLL CHAOTIC SYSTEMS

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ABSTRACT

This paper discusses the design of active controllers for generalized projective synchronization (GPS) of identical Wang 3-scroll chaotic systems (Wang, 2009), identical Dadras 3-scroll chaotic systems (Dadras and Momeni, 2009) and non-identical Wang 3-scroll system and Dadras 3-scroll system. The synchronization results (GPS) derived in this paper for the 3-scroll chaotic systems have been derived using active control method and established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active control method is very effective and convenient for achieving the generalized projective synchronization (GPS) of the 3-scroll chaotic systems addressed in this paper. Numerical simulations are provided to illustrate the effectiveness of the GPS synchronization results derived in this paper.

KEYWORDS

Active Control, Chaos, Chaotic Systems, Generalized Projective Synchronization, 3-Scroll Systems.

1. INTRODUCTION

Chaotic systems are nonlinear dynamical systems which are highly sensitive to initial conditions. This sensitivity of chaotic systems is usually called as the *butterfly effect* [1]. Experimentally, chaos was first discovered by Lorenz ([2], 1963) while he was simulating weather models. A chaotic system simpler than the Lorenz system was proposed by Rössler ([3], 1976). The theoretical equations of the Rössler system were later found to be useful in modelling equilibrium in chemical reactions.

Chaos synchronization problem received great attention in the literature when Pecora and Carroll [4] published their results on chaos synchronization in 1990. From then on, chaos synchronization has been extensively and intensively studied in the last three decades [4-35]. Chaos theory has been explored in a variety of fields including physical systems [5], chemical systems [6], ecological systems [7], secure communications [8-10], etc.

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Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically. In other words, the sum of the states of the master and slave systems are designed to converge to zero asymptotically, when anti-synchronization appears.

In the recent years, various schemes have been deployed for chaos synchronization such as PC method [4], OGY method [11], active control [12-15], adaptive control [16-20], backstepping design [21-23], sampled-data feedback [24], sliding mode control [25-28], etc.

In generalized projective synchronization (GPS) of chaotic systems [29-30], the chaotic systems can synchronize up to a constant scaling matrix. Complete synchronization [12-13], anti-synchronization [31-34], hybrid synchronization [35], projective synchronization [36] and generalized synchronization [37] are particular cases of generalized projective synchronization. GPS has important applications in areas like secure communications and secure data encryption.

In this paper, we deploy active control method so as to derive new results for the generalized projective synchronization (GPS) for identical and different Wang 3-scroll systems and Dadras 3-scroll chaotic systems. Explicitly, using active nonlinear control and Lyapunov stability theory, we achieve generalized projective synchronization for identical Wang 3-scroll chaotic systems (Wang, [38], 2009), identical Dadras 3-scroll chaotic systems (Dadras and Momeni, [39], 2009) and non-identical Wang 3-scroll system and Dadras 3-scroll system.

This paper has been organized as follows. In Section 2, we give the problem statement and our methodology. In Section 3, we present a description of the 3-scroll chaotic systems considered in this paper. In Section 4, we derive results for the GPS of two identical Wang 3-scroll chaotic systems. In Section 5, we derive results for the GPS of two identical Dadras 3-scroll chaotic systems. In Section 6, we discuss the GPS of non-identical 3-scroll chaotic systems. In Section 7, we summarize the main results derived in this paper.

2. PROBLEM STATEMENT AND OUR METHODOLOGY

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system, B is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ is the nonlinear part of the system and $u \in R^n$ is the controller of the slave system.

If $A = B$ and $f = g$, then x and y are the states of two identical chaotic systems. If $A \neq B$ or $f \neq g$, then x and y are the states of two different chaotic systems.

In the active control approach, we design a feedback controller u, which achieves the generalized projective synchronization (GPS) between the states of the master system (1) and the slave system (2) for all initial conditions $x(0), z(0) \in R^n$.

For the GPS of the systems (1) and (2), the *synchronization error* is defined as

$$e = y - Mx, \tag{3}$$

where

$$M = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_n \end{bmatrix} \tag{4}$$

In other words, we have

$$e_i = y_i - \alpha_i x_i, \quad (i = 1, 2, \ldots, n) \tag{5}$$

From (1)-(3), the error dynamics is easily obtained as

$$\dot{e} = By - MAx + g(y) - Mf(x) + u \tag{6}$$

The aim of GPS is to find a feedback controller u so that

$$\lim_{t \to \infty} \|e(t)\| = 0 \text{ for all } e(0) \in R^n. \tag{7}$$

Thus, the problem of generalized projective synchronization (GPS) between the master system (1) and slave system (2) can be translated into a problem of how to realize the asymptotic stabilization of the system (6). So, the objective is to design an active controller u for stabilizing the error dynamical system (6) at the origin.

We take as a candidate Lyapunov function

$$V(e) = e^T P e, \tag{8}$$

where P is a positive definite matrix.

Note that $V : R^n \to R$ is a positive definite function by construction.

We assume that the parameters of the master and slave system are known and that the states of both systems (1) and (2) are measurable.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{9}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [40], the error dynamics (6) is globally exponentially stable and hence the condition (7) will be satisfied. Hence, GPS is achieved between the states of the master system (1) and the slave system (2).

3. SYSTEMS DESCRIPTION

The Wang 3-scroll system ([38], 2009) is described by the dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_1 - x_2) - x_2 x_3 \\ \dot{x}_2 &= -b x_2 + x_1 x_3 \\ \dot{x}_3 &= -c x_3 + d x_1 + x_1 x_2\end{aligned} \tag{10}$$

where x_1, x_2, x_3 are the state variables and a, b, c, d are constant, positive parameters of the system.

The Wang dynamics (10) exhibits a 3-scroll chaotic attractor when the system parameter values are chosen as

$$a = 0.977, \quad b = 10, \quad c = 4, \quad d = 0.1$$

Figure 1 depicts the strange attractor of the Wang 3-scroll chaotic system.

The Dadras 3-scroll system ([39], 2009) is described by the dynamics

$$\begin{aligned}\dot{x}_1 &= x_2 - p x_1 + q x_2 x_3 \\ \dot{x}_2 &= r x_2 - x_1 x_3 + x_3 \\ \dot{x}_3 &= s x_1 x_2 - \varepsilon x_3\end{aligned} \tag{11}$$

where x_1, x_2, x_3 are the state variables and p, q, r, s, ε are constant, positive parameters of the system.

The Dadras dynamics (11) is chaotic when the parameter values are taken as

$$p = 3, \quad q = 2.7, \quad r = 1.7, \quad s = 2, \quad \varepsilon = 9$$

Figure 2 describes the strange attractor of the Dadras 3-scroll chaotic system (11).

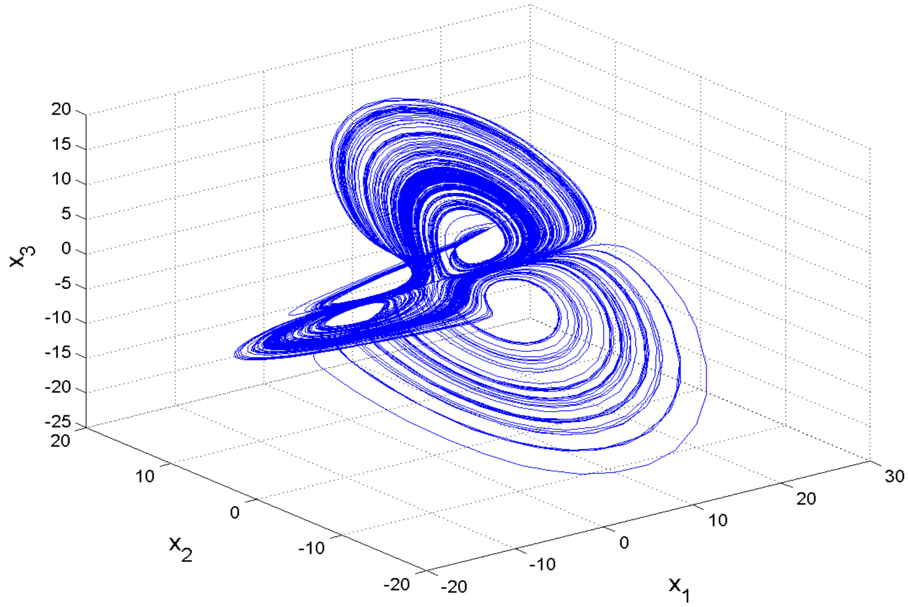

Figure 1. Strange Attractor of the Wang 3-Scroll Chaotic System

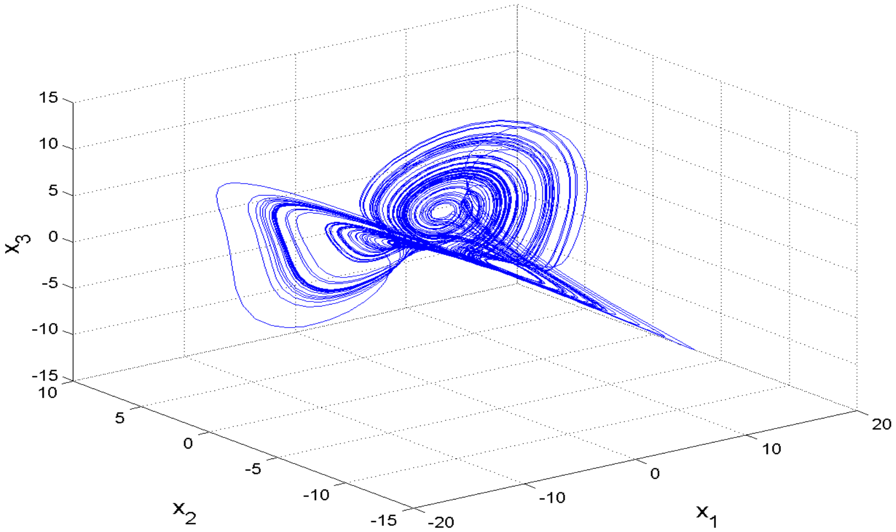

Figure 2. Strange Attractor of the Dadras 3-Scroll Chaotic System

4. GPS OF IDENTICAL WANG 3-SCROLL CHAOTIC SYSTEMS

4.1 Theoretical Results

In this section, we apply the active nonlinear control method for the generalized projective synchronization (GPS) of two identical Wang 3-scroll chaotic systems ([38], 2009).
Thus, the master system is described by the Wang dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_1 - x_2) - x_2 x_3 \\ \dot{x}_2 &= -b x_2 + x_1 x_3 \\ \dot{x}_3 &= -c x_3 + d x_1 + x_1 x_2 \end{aligned} \tag{12}$$

where x_1, x_2, x_3 are the states and a, b, c, d are positive, constant parameters of the system.
The slave system is described by the controlled Wang dynamics

$$\begin{aligned} \dot{y}_1 &= a(y_1 - y_2) - y_2 y_3 + u_1 \\ \dot{y}_2 &= -b y_2 + y_1 y_3 + u_2 \\ \dot{y}_3 &= -c y_3 + d y_1 + y_1 y_2 + u_3 \end{aligned} \tag{13}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the active nonlinear controls to be designed.
For the GPS of the Wang systems (12) and (13), the synchronization error e is defined by

$$\begin{aligned} e_1 &= y_1 - \alpha_1 x_1 \\ e_2 &= y_2 - \alpha_2 x_2 \\ e_3 &= y_3 - \alpha_3 x_3 \end{aligned} \tag{14}$$

where the scales $\alpha_1, \alpha_2, \alpha_3$ are real numbers.

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a e_1 - a(y_2 - \alpha_1 x_2) - y_2 y_3 + \alpha_1 x_2 x_3 + u_1 \\ \dot{e}_2 &= -b e_2 + y_1 y_3 - \alpha_2 x_1 x_3 + u_2 \\ \dot{e}_3 &= -c e_3 + d(y_1 - \alpha_3 x_1) + y_1 y_2 - \alpha_3 x_1 x_2 + u_3 \end{aligned} \tag{15}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= -a e_1 + a(y_2 - \alpha_1 x_2) + y_2 y_3 - \alpha_1 x_2 x_3 - k_1 e_1 \\ u_2 &= b e_2 - y_1 y_3 + \alpha_2 x_1 x_3 - k_2 e_2 \\ u_3 &= c e_3 - d(y_1 - \alpha_3 x_1) - y_1 y_2 + \alpha_3 x_1 x_2 - k_3 e_3 \end{aligned} \tag{16}$$

where the gains k_1, k_2, k_3 are positive constants.

Substituting (16) into (15), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= -k_1 e_1\\ \dot{e}_2 &= -k_2 e_2\\ \dot{e}_3 &= -k_3 e_3\end{aligned} \tag{17}$$

Next, we prove the following result.

Theorem 1. The active feedback controller (16) achieves global chaos generalized projective synchronization (GPS) between the identical Wang 3-scroll chaotic systems (12) and (13).

Proof. We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{18}$$

which is a positive definite function on R^3.

Differentiating (18) along the trajectories of (17), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \tag{19}$$

which is a negative definite function on R^3.

Thus, by Lyapunov stability theory [40], the error dynamics (17) is globally exponentially stable.

This completes the proof. ■

4.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the two systems of differential equations (12) and (13) with the active controller (16).

The parameters of the identical Wang 3-scroll chaotic systems are chosen as

$$a = 0.977, \quad b = 10, \quad c = 4, \quad d = 0.1$$

The initial values for the master system (12) are taken as

$$x_1(0) = 16, \quad x_2(0) = -7, \quad x_3(0) = 20$$

The initial values for the slave system (13) are taken as

$$y_1(0) = 4, \quad y_2(0) = 12, \quad y_3(0) = -9$$

The GPS scales are taken as $\alpha_1 = -2.7$, $\alpha_2 = 1.7$ and $\alpha_3 = -1.4$.

We take the state feedback gains as $k_i = 5$ for $i = 1, 2, 3$.

Figure 3 shows the GPS synchronization of the identical Wang 3-scroll systems. Figure 4 shows the time-history of the GPS synchronization errors e_1, e_2, e_3 for the identical Wang 3-scroll systems.

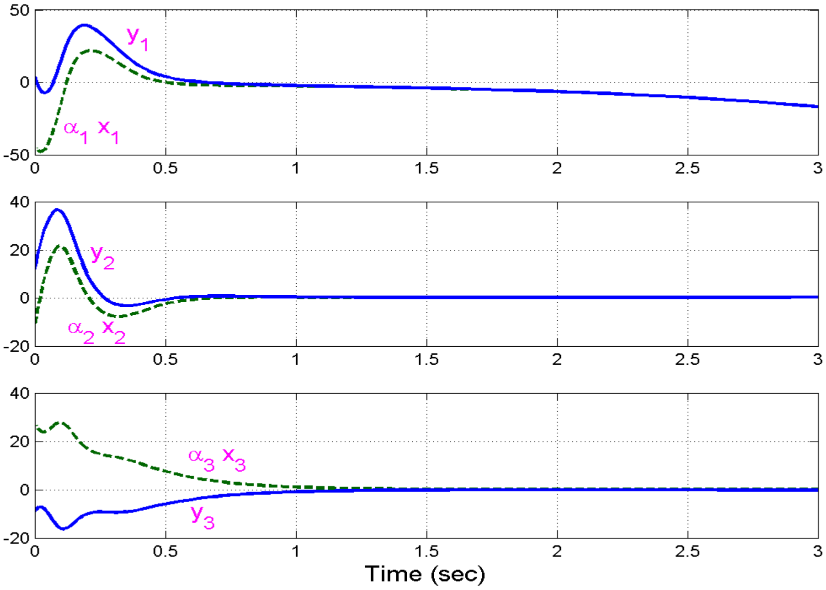

Figure 3. GPS Synchronization of the Identical Wang 3-Scroll Systems

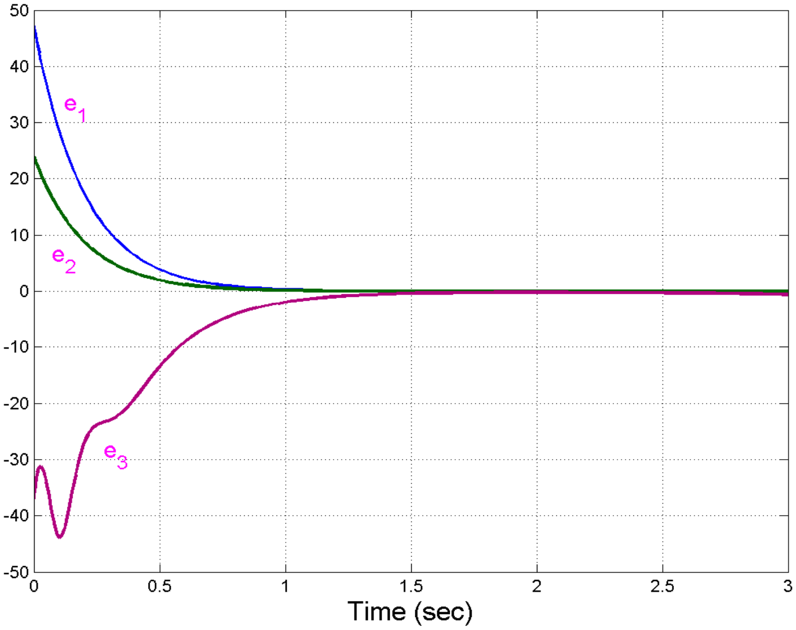

Figure 4. Time History of the GPS Synchronization Error

5. GPS OF IDENTICAL DADRAS 3-SCROLL CHAOTIC SYSTEMS

5.1 Theoretical Results

In this section, we apply the active nonlinear control method for the generalized projective synchronization (GPS) of two identical Dadras 3-scroll chaotic systems ([39], 2009).
Thus, the master system is described by the Dadras dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 - px_1 + qx_2x_3 \\ \dot{x}_2 &= rx_2 - x_1x_3 + x_3 \\ \dot{x}_3 &= sx_1x_2 - \varepsilon x_3 \end{aligned} \tag{20}$$

where x_1, x_2, x_3 are the states and p, q, r, s, ε are positive, constant parameters of the system.

The slave system is described by the controlled Dadras dynamics

$$\begin{aligned} \dot{y}_1 &= y_2 - py_1 + qy_2y_3 + u_1 \\ \dot{y}_2 &= ry_2 - y_1y_3 + y_3 + u_2 \\ \dot{y}_3 &= sy_1y_2 - \varepsilon y_3 + u_3 \end{aligned} \tag{21}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the active nonlinear controls to be designed.

For the GPS of the Dadras systems (20) and (21), the synchronization error e is defined by

$$\begin{aligned} e_1 &= y_1 - \alpha_1 x_1 \\ e_2 &= y_2 - \alpha_2 x_2 \\ e_3 &= y_3 - \alpha_3 x_3 \end{aligned} \tag{22}$$

where the scales $\alpha_1, \alpha_2, \alpha_3$ are real numbers.

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= -pe_1 + y_2 - \alpha_1 x_2 + q(y_2y_3 - \alpha_1 x_2x_3) + u_1 \\ \dot{e}_2 &= re_2 + y_3 - \alpha_2 x_3 - y_1y_3 + \alpha_2 x_1x_3 + u_2 \\ \dot{e}_3 &= -\varepsilon e_3 + s(y_1y_2 - \alpha_3 x_1x_2) + u_3 \end{aligned} \tag{23}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= pe_1 - y_2 + \alpha_1 x_2 - q(y_2y_3 - \alpha_1 x_2x_3) - k_1e_1 \\ u_2 &= -re_2 - y_3 + \alpha_2 x_3 + y_1y_3 - \alpha_2 x_1x_3 - k_2e_2 \\ u_3 &= \varepsilon e_3 - s(y_1y_2 - \alpha_3 x_1x_2) - k_3e_3 \end{aligned} \tag{24}$$

where the gains k_1, k_2, k_3 are positive constants.

Substituting (24) into (23), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \end{aligned} \tag{25}$$

Next, we prove the following result.

Theorem 2. The active feedback controller (24) achieves global chaos generalized projective synchronization (GPS) between the identical Dadras 3-scroll chaotic systems (20) and (21).

Proof. We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{26}$$

which is a positive definite function on R^3.

Differentiating (26) along the trajectories of (25), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \tag{27}$$

which is a negative definite function on R^3.

Thus, by Lyapunov stability theory [40], the error dynamics (25) is globally exponentially stable.

This completes the proof. ■

5.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the two systems of differential equations (20) and (21) with the active controller (24).
The parameters of the identical Dadras 3-scroll chaotic systems are chosen as

$$p = 3, \;\; q = 2.7, \;\; r = 1.7, \;\; s = 2, \;\; \varepsilon = 9$$

The initial values for the master system (20) are taken as

$$x_1(0) = -5, \;\; x_2(0) = 26, \;\; x_3(0) = 14$$

The initial values for the slave system (21) are taken as

$$y_1(0) = 24, \;\; y_2(0) = 2, \;\; y_3(0) = -6$$

The GPS scales are taken as $\alpha_1 = 3.1$, $\alpha_2 = 2.4$ and $\alpha_3 = 1.6$.

We take the state feedback gains as $k_i = 5$ for $i = 1, 2, 3$.

Figure 5 shows the GPS synchronization of the identical Dadras 3-scroll systems. Figure 6 shows the time-history of the GPS synchronization errors e_1, e_2, e_3 for the identical Dadras 3-scroll systems.

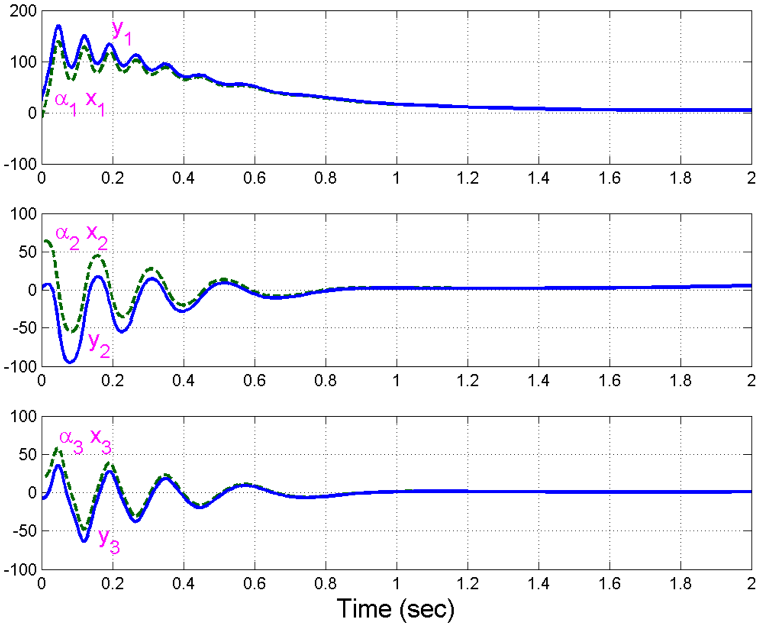

Figure 5. GPS Synchronization of the Identical Dadras 3-Scroll Systems

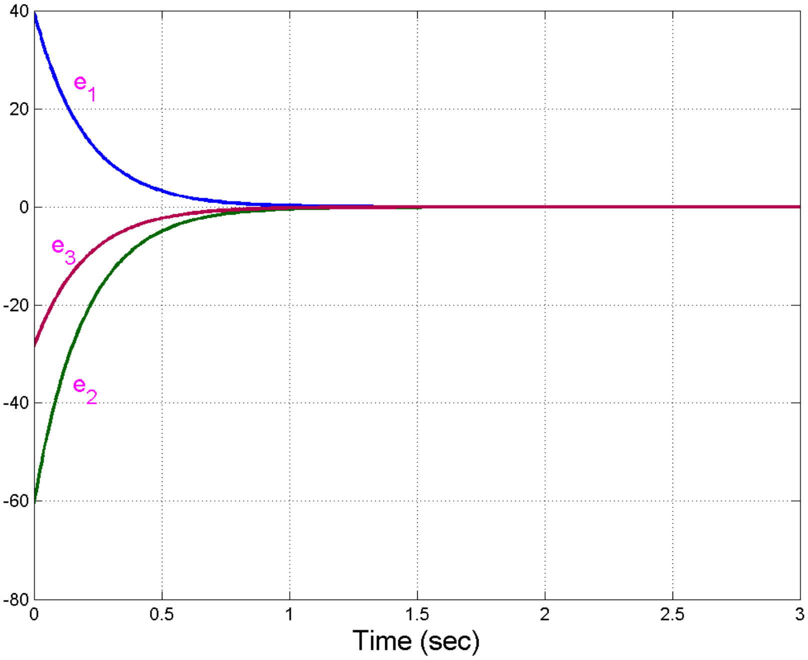

Figure 6. Time History of the GPS Synchronization Error

6. GPS OF WANG 3-SCROLL AND DADRAS 3-SCROLL SYSTEMS

6.1 Theoretical Results

In this section, we apply the active nonlinear control method for the generalized projective synchronization (GPS) of Wang 3-scroll and Dadras 3-scroll chaotic systems.
Thus, the master system is described by the Wang dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_1 - x_2) - x_2 x_3 \\ \dot{x}_2 &= -bx_2 + x_1 x_3 \\ \dot{x}_3 &= -cx_3 + dx_1 + x_1 x_2 \end{aligned} \tag{28}$$

where x_1, x_2, x_3 are the states and a, b, c, d are constant, positive parameters of the system.

The slave system is described by the controlled Dadras dynamics

$$\begin{aligned} \dot{y}_1 &= y_2 - py_1 + qy_2 y_3 + u_1 \\ \dot{y}_2 &= ry_2 - y_1 y_3 + y_3 + u_2 \\ \dot{y}_3 &= sy_1 y_2 - \varepsilon y_3 + u_3 \end{aligned} \tag{29}$$

where y_1, y_2, y_3 are the states, p, q, r, s, ε are positive, constant parameters of the system and u_1, u_2, u_3 are the active nonlinear controls to be designed.

For the GPS of the 3-scroll systems (28) and (29), the synchronization error e is defined by

$$e_i = y_i - \alpha_i x_i, \quad (i = 1, 2, 3) \tag{30}$$

where the scales $\alpha_1, \alpha_2, \alpha_3$ are real numbers.

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= y_2 - py_1 + qy_2 y_3 - \alpha_1 \left[a(x_1 - x_2) - x_2 x_3 \right] + u_1 \\ \dot{e}_2 &= ry_2 - y_1 y_3 + y_3 - \alpha_2 \left[-bx_2 + x_1 x_3 \right] + u_2 \\ \dot{e}_3 &= sy_1 y_2 - \varepsilon y_3 - \alpha_3 \left[-cx_3 + dx_1 + x_1 x_2 \right] + u_3 \end{aligned} \tag{31}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= -y_2 + py_1 - qy_2 y_3 + \alpha_1 \left[a(x_1 - x_2) - x_2 x_3 \right] - k_1 e_1 \\ u_2 &= -ry_2 + y_1 y_3 - y_3 + \alpha_2 \left[-bx_2 + x_1 x_3 \right] - k_2 e_2 \\ u_3 &= -sy_1 y_2 + \varepsilon y_3 + \alpha_3 \left[-cx_3 + dx_1 + x_1 x_2 \right] - k_3 e_3 \end{aligned} \tag{32}$$

where the gains k_1, k_2, k_3 are positive constants.

Substituting (32) into (31), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \end{aligned} \tag{33}$$

Next, we prove the following result.

Theorem 3. The active feedback controller (32) achieves global chaos generalized projective synchronization (GPS) between the Wang 3-scroll system (28) and Dadras 3-scroll system (29).

Proof. We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{34}$$

which is a positive definite function on R^3.

Differentiating (26) along the trajectories of (33), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \tag{35}$$

which is a negative definite function on R^3.

Thus, by Lyapunov stability theory [40], the error dynamics (33) is globally exponentially stable. This completes the proof. ■

6.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the two systems of differential equations (28) and (29) with the active controller (32).
The parameters of the Wang 3-scroll and Dadras 3-scroll systems are chosen as

$$a = 0.977,\ \ b = 10,\ \ c = 4,\ \ d = 0.1,\ p = 3,\ \ q = 2.7,\ r = 1.7,\ \ s = 2,\ \ \varepsilon = 9$$

The initial values for the master system (28) are taken as

$$x_1(0) = 23,\ \ x_2(0) = -4,\ \ x_3(0) = -9$$

The initial values for the slave system (29) are taken as

$$y_1(0) = -7,\ \ y_2(0) = 12,\ \ y_3(0) = 20$$

The GPS scales are taken as $\alpha_1 = 2.7,\ \ \alpha_2 = -1.2$ and $\alpha_3 = 5.6$.

We take the state feedback gains as $k_i = 5$ for $i = 1, 2, 3$.

Figure 7 shows the GPS synchronization of the non-identical Wang and Dadras 3-scroll systems. Figure 8 shows the time-history of the GPS synchronization errors e_1, e_2, e_3 for the non-identical Wang and Dadras 3-scroll systems.

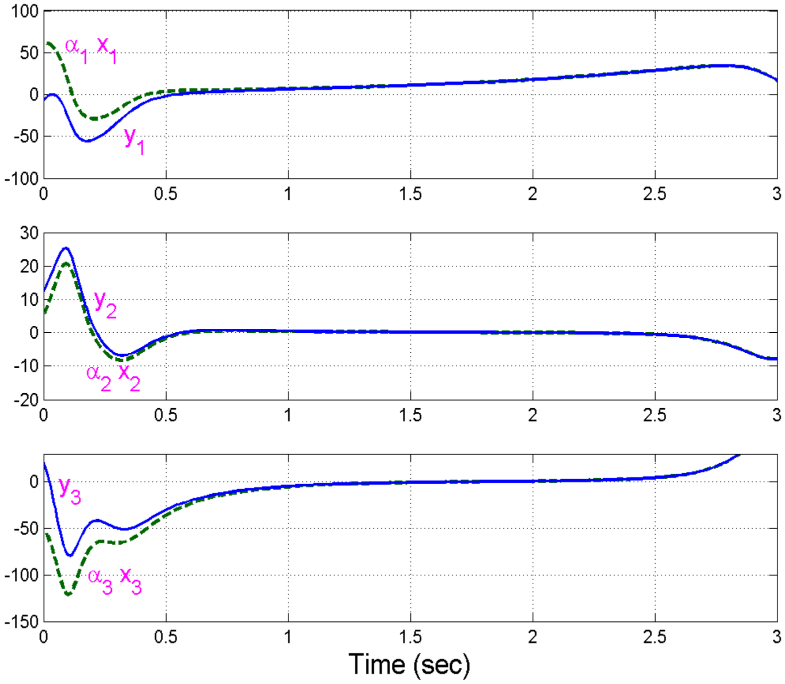

Figure 7. GPS Synchronization of the Wang and Dadras 3-Scroll Systems

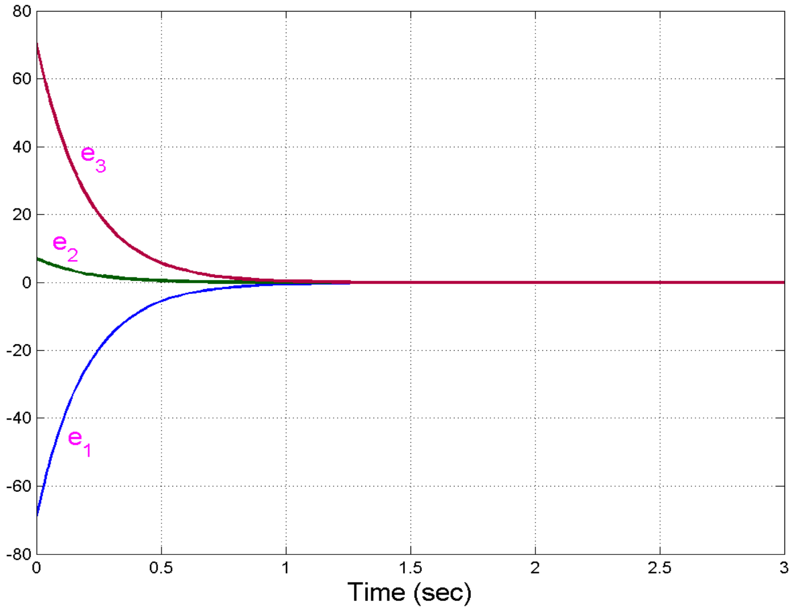

Figure 8. Time History of the GPS Synchronization Error

7. CONCLUSIONS

In this paper, we had derived active control laws for achieving generalized projective synchronization (GPS) of the following pairs of 3-scroll chaotic systems:

(A) Identical Wang 3-scroll systems (2009)
(B) Identical Dadras 3-scroll systems (2009)
(C) Non-identical Wang and Dadras 3-scroll systems

The synchronization results (GPS) derived in this paper for the Wang and Dadras 3-scroll systems have been proved using Lyapunov stability theory. Since Lyapunov exponents are not required for these calculations, the proposed active control method is very effective and suitable for achieving GPS of the 3-scroll chaotic systems addressed in this paper. Numerical simulations are shown to demonstrate the effectiveness of the GPS synchronization results derived in this paper for the Wang and Dadras 3-scroll chaotic systems.

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